

# **FLUX FROM A LARGE HEATED WALL**

**by**

**ALAKANANDA BANDYOPADHYAY**



**DEPARTMENT OF MECHANICAL ENGINEERING  
INSTITUTE OF TECHNOLOGY, KAT**

**FEBRUARY, 1987**

CONJUGATE HEAT TRANSFER IN THE LAMINAR FLOW OF HIGH  
PRANDTL NUMBER FLUIDS IN A CIRCULAR TUBE SUBJECTED  
TO NON-UNIFORM CIRCUMFERENTIAL RADIATION HEAT  
FLUX FROM A LARGE HEATED WALL

A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of

**MASTER OF TECHNOLOGY**

by  
**ALAKANANDA BANDYOPADHYAY**

*to the*

**DEPARTMENT OF MECHANICAL ENGINEERING**

**INDIAN INSTITUTE OF TECHNOLOGY, KANPUR**

**FEBRUARY, 1987**

CENTRAL LIBRARY

Acc. No. 98464

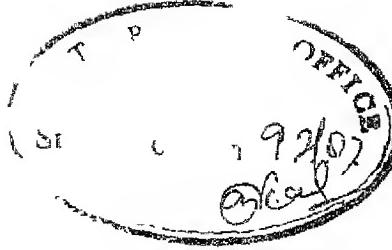
ME-1987-M-BAN-CON

Thesis  
62.112  
B2.23e

*In*

*Loving Memory of*  
*My Father*

CERTIFICATE



This is to certify that the work presented in this thesis entitled "CONJUGATE HEAT TRANSFER IN THE LAMINAR FLOW OF HIGH PRANDTL NUMBER FLUIDS IN A CIRCULAR TUBE SUBJECTED TO NON-UNIFORM CIRCUMFERENTIAL RADIATION HEAT FLUX FROM A LARGE HEATED WALL" has been carried out by Mr. Alakananda Bandyopadhyay under my supervision and has not been submitted elsewhere for a degree.

Dr. P.S. Ghoshdastidar  
Assistant Professor  
Department of Mechanical Engineering  
Indian Institute of Technology  
Kanpur

February 1987

## ACKNOWLEDGEMENTS

I take this opportunity to express my deep sense of gratitude and profound regard to Dr. P.S. Ghoshdastidar for his invaluable guidance and constant encouragement which was vital to the success of this effort.

I want to express my thanks to Mr. Aniruddha Mukhopadhyay for his constant help particularly in computational work.

Thanks are also due to Mr. R.N. Srivastava for his neat and elegant typing and Mr. A. Ganguly for the fine drawing work.

ALAKANANDA BANDYOPADHYAY

CONTENTS

	Page
LIST OF FIGURES	vi
NOMENCLATURE	viii
ABSTRACT	x
CHAPTER 1 INTRODUCTION	1
1.1 General	1
1.2 Objective	2
1.3 Literature Survey	2
1.4 Salient Features of the Present Investigation	3
CHAPTER 2 PROBLEM FORMULATION	5
2.1 Description of the Problem	5
2.2 Basic Equations	8
2.3 Fluid Bulk Temperature	10
CHAPTER 3 METHOD OF SOLUTION	13
3.1 Finite Difference Approximations	13
3.1.1 Discretization of the Energy Equation for Solid	15
3.1.2 Discretization of the Energy Equation for Fluid	20
3.1.3 Discretization of the Compatibility Conditions at the Fluid-Solid Interface	22
3.1.4 Simplified Equations for $T_{fb}$ and $q_i(x, \theta)$	26
3.2 Numerical Solution of the Discretized Equations	27
3.3 Algorithm for the Numerical Solution	30
CHAPTER 4 RESULTS AND DISCUSSIONS	33
CHAPTER 5 CONCLUSIONS	47
REFERENCES	48
APPENDIX A DETERMINATION OF THE CONFIGURATION FACTOR	49
A.1 Definition	49
A.2 Hottel's Cross-String Method	49
A.3 Derivation of the Equations for Determining Configuration Factor	51

	Page
APPENDIX B THE LINE-BY-LINE METHOD	64
APPENDIX C INPUT DATA TO THE PROGRAM	67
APPENDIX D LISTING OF THE COMPUTER PROGRAM	69

## LIST OF FIGURES

### Title

Schematic diagram of the problem and coordinate system in  $(r, \theta)$  direction

Schematic diagram of the problem and coordinate system in  $(r, x)$  direction

A typical finite difference grid system

Finite difference nodes near and at the interface

A flow chart of the computer program

Radial variation of the temperature in the tube wall for various thermal conductivities at  $\theta = 0^\circ$

Temperature variation in the tube wall for different thickness of the tube

Radial temperature variation in the tube wall for different vertical position ( $d$ ) of the tube with respect to the heated wall

Circumferential variation of the fluid-solid interface temperature for different widths of the heated wall

Axial distribution of mean wall temperature for different Prandtl number of the fluid, Re being constant

Axial distribution of the local Nusselt number for different values of Prandtl number of the fluid

Axial distribution of the local Nusselt number for different values of Reynolds number of the flow

Axial distribution of fluid bulk temperature for different Reynolds number of the flow

Axial distribution of fluid bulk temperature for different values of Prandtl number of the fluid

Determination of configuration factor between two surfaces by Hottel's Cross-String method

Number	Title	Page
A-2		52
A-3		57
A-4		59
A-5		61
B-1	Representation of the line-by-line method	65

NOMENCLATURE

## Symbols:

- $C$  = specific heat of the fluid  
 $d$  = vertical distance of the tube from the heated wall  
 $(\text{Figure } 1(\text{a}))$   
 $F$  = configuration factor  
 $h$  = convective heat transfer coefficient at the solid-fluid interface  
 $k$  = thermal conductivity  
 $L$  = horizontal distance of the centre of the tube from the left end of the heated wall ( $\text{Figure } 1(\text{a})$ )  
 $N$  = total number of grid points  
 $\text{Nu}$  = Nusselt number  
 $\text{Pr}$  = Prandtl number  
 $q$  = heat flux  
 $r$  = radial coordinate  
 $\Delta r$  = step size in  $r$ -direction  
 $\text{Re}$  = Reynolds number  
 $T$  = temperature  
 $\text{Th}$  = thickness of the tube wall  
 $u$  = axial velocity of the fluid  
 $w$  = width of the heated wall ( $\text{Figure } 1(\text{a})$ )  
 $x$  = axial distance  
 $\Delta x$  = step size in  $x$ -direction

## Greek letters:

- $\alpha$  = thermal diffusivity of the fluid  
 $\epsilon$  = emissivity of the tube material  
 $\mu$  = coefficient of viscosity of the fluid

$\nu$  = coefficient of kinematic viscosity of the fluid  
 $\rho$  = density of the fluid  
 $\sigma$  = Stefan-Boltzmann constant  
 $\phi$  = circumferential coordinate  
 $\Delta\phi$  = step size in  $\phi$ -direction

Subscripts:

$_{fb}$  = fluid bulk  
 $e$  = at the entrance of the tube  
 $f$  = fluid  
 $i$  = index notation for a node in r-direction  
 $j$  = index notation for a node in  $\phi$ -direction  
 $k$  = index notation for a grid in x-direction  
 $p$  = value at previous x-station  
 $p$  = at constant pressure  
 $r$  = along r-direction, entirely in the fluid and wall of the tube  
 $rf$  = along r-direction, only in fluid region  
 $s$  = surrounding medium  
 $s$  = solid  
 $w$  = heated wall  
 $x$  = local value

Superscripts:

$-$  = average value  
 $*$  = non-dimensional value  
 $n$  = value at the previous iteration.

## ABSTRACT

A simple but computationally efficient finite difference scheme has been used to solve the problem of conjugate heat transfer in a laminar flow of high Prandtl number fluid in a circular tube whose periphery is exposed to non-uniform radiation heat flux from a large heated wall. The present model is a simulation of the effect of hot refractory wall close to a tube in a pipe still.

The results indicate that in a thin tube with low conductivity, the temperature of the tube is very high which may result in hot spots on the tube surface. If the tube material is thick and has a sufficiently high conductivity, the peripheral conduction in the tube wall alleviates this problem. Also, the temperature of the tube goes up when the tube is placed closer to the heated wall.

For the wider wall the mean temperature of the inner wall of the tube is more as the tube receives more radiation. Also, it is observed that mean tube inner wall temperature increases with the axial distance and is lower for higher Prandtl number fluid.

It is also seen that thermal entry lengths are higher for higher Prandtl number fluid and for higher Reynolds number of the flow.

The fluid bulk temperature increases with the axial distance and for higher Prandtl number of the fluid and lower Reynolds number of the flow, the fluid bulk temperatures are less.

The accuracy of the present numerical solution has been checked by comparing it with the analytical solution of the case of fully developed constant heat flux condition on the periphery of a very thin tube. The result is found to be very satisfactory.

## CHAPTER 1

### INTRODUCTION

#### 1.1 General

This investigation presents the case of heating of a fluid flowing through a long circular tube which is exposed to a non-uniform radiation heat flux on its periphery by the proximity of a very large heated wall situated parallelly above the axis of the tube. The temperature of the fluid is uniform at the inlet of the tube where the heat transfer begins, but the velocity profile is already fully developed and invariant. This, however, limits the applicability of the present analysis to fluids whose Prandtl numbers are very high relative to 1, for example, oils. The present analysis is for constant property fluids but variable property fluids can also be investigated with little modification in the solution procedure.

The present model is a simulation of the effect of a refractory wall close to a tube in a pipe still. Frequently, in the case of uneven distribution of heat flux around the tube, hot spots on the tube surface may occur. If the tube material is thick and has a sufficiently high conductivity, peripheral conduction in the wall may alleviate this problem, but for thin walled tubes, the difficulty can be acute.

The temperature distribution in the wall is a strong function of the amount of radiation heat flux received at the outer surface of the tube and the heat transfer at the inter-

between the inner surface of the tube and the fluid. Thus, the conjugate heat transfer at the interface plays an important role in this problem.

### 1.2 Objective

The objectives of the present investigation are (a) to find the temperature distribution in the wall of the tube for various thickness and thermal conductivity of the tube and for various positions of the tube with respect to the heated wall, (b) to get a qualitative indication of the thermal entry length for fluids of different Prandtl numbers and for various Reynolds number of the flow, and (c) to show circumferential variation of solid-fluid interface temperatures for different widths of the heated wall.

### 1.3 Literature Survey

The problem of laminar flow in a circular tube with fully developed constant heat rate conditions axially, but with an arbitrary peripheral variation of heat flux, has been treated by Reynolds [1, 2]. In [1], a solution is obtained for a pulse of heat transferred across an elemental section of wall, and superposition is employed to obtain solutions for any prescribed heat flux. In [2], the prescribed heat flux is expressed in terms of a Fourier expansion and also the flow is turbulent. An interesting conclusion of [2] is that the effects of circumferential heat flux variation in turbulent flow are sometimes more pronounced than in laminar flow. The coupled effect of wall and fluid conduction in laminar pipe flow has been studied

by Faghri and Sparrow [3] and Zariffch et al [4] using finite difference and by Campo and Rangel [5] analytically. Davis and Venkatesh [6] also studied heat transfer to Poiseuille flow in a pipe with heat conducting section, as an example of their new general formulation of the solution of conjugate boundary value problem. Computations, however, were not performed. Luikov et al [7] presented a closed form analytical solution for the conjugate heat transfer problem in case of fluid flow in circular tube. S.V. Patankar [8] suggested the use of harmonic mean of the transport properties on both side of an interface to treat the points of discontinuity. Thermally fully developed flow in a square duct with finite wall thickness was presented as an example. The method is quite general and can also calculate fluid flow and heat transfer in arbitrary geometries. Barozzi and Pagiliarini [9] have used finite element method to solve axi-symmetric conjugate heat transfer in a pipe with uniform heat flux specified on its periphery. The method combines a finite element treatment of the energy equation in the tube wall and the application of Duhamel's theorem at the interface.

#### 1.4 Salient Features of the Present Investigation

The present investigation differs from the previous works on the following counts:

- (i) the non-uniform peripheral heat flux on the outer surface of the tube is actually calculated, instead of being prescribed.

- (ii) the flow is thermally undeveloped.
- (iii) the problem is non-axisymmetric.
- (iv) a simple but computationally efficient finite difference scheme has been used to solve the problem.
- (v) it has the capability of handling variable property fluid and tube material with little modification in the numerical method.

## CHAPTER 2

### PROBLEM FORMULATION

#### 2.1 Description of the Problem

The physical model is depicted in Figure 1(a) and Figure 1(b). The refractory wall has a width  $W$ . The pipe, which has inside and outside radii  $r_1$  and  $r_2$ , is separated by a distance  $d$  vertically from the wall, the tube being at a distance  $L$  from the left extreme point of the wall.  $L$  and  $d$  together determine the position of the tube with respect to the wall. The wall behaves as a black body radiator at temperature  $T_w$ . The surrounding is at temperature  $T_s$ . The upper portion of the periphery of the tube receives radiation heat flux both from the wall and the surroundings while the bottom portion receives radiation heat flux primarily from the surroundings. So, there is non-uniform radiation heat flux on the periphery of the tube (Figure 1(a)), but the radiation heat flux is uniform in the axial direction since both the wall and the tube are very long in that direction (Figure 1(b)). A fluid enters the tube at a uniform inlet temperature  $T_e$  while the velocity profile is fully developed.

So, the thermal boundary layer develops along the axis of the tube until the temperature profile is fully developed. The local heat flux density at any  $x$  position from the inner pipe wall to the fluid is,

$$q_1 = q_1(\emptyset) = h(T_1 - T_{fb})$$

where  $T_{fb}$  is the fluid bulk temperature at that  $x$ -position and  $h$  is the heat transfer coefficient at the given  $x$  (the

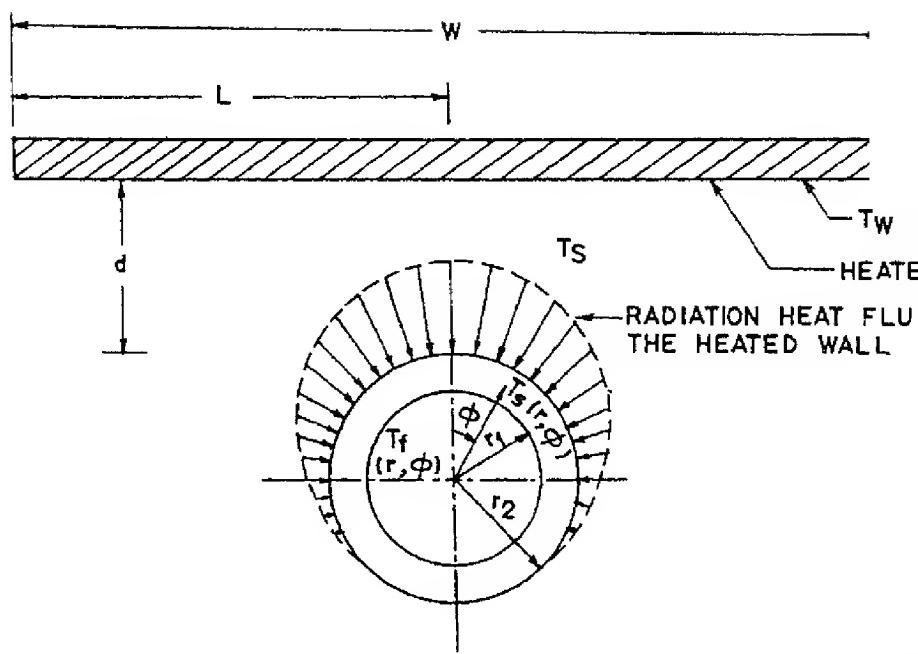


FIG. 1(a) SCHEMATIC DIAGRAM OF THE PROBLEM AND THE COORDINATE SYSTEM IN  $(r, \phi)$  DIRECTION

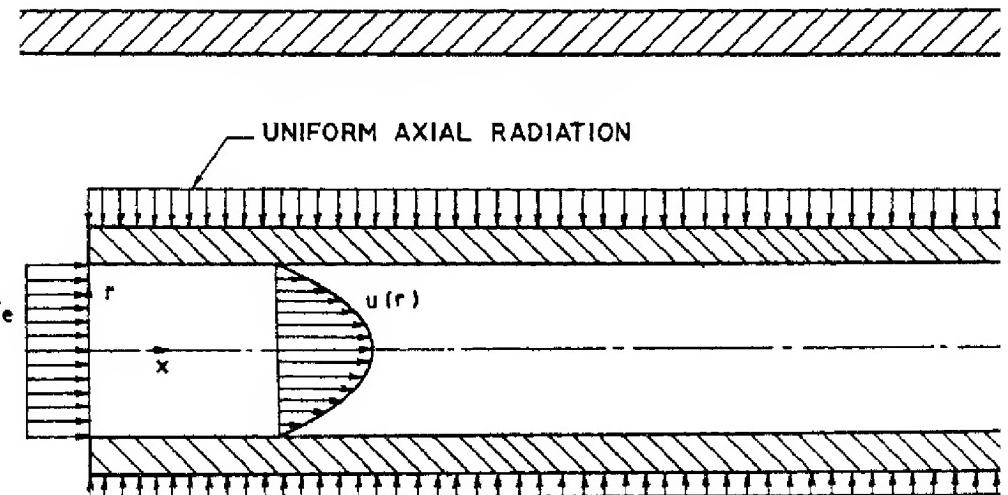


FIG. 1(b) SCHEMATIC DIAGRAM OF THE PROBLEM AND THE COORDINATE SYSTEM IN  $(r, x)$  DIRECTION

method of determination of their values shown later).  $T_1$  is the wall temperature at same x-station. The polar coordinates ( $r, \theta$ ) are shown in Figure 1(a).

The corresponding radiant flux density to a point on the outer surface where the temperature  $T_2 = T_2(\theta)$  is,

$$q_2 = q_2(\theta) = \sigma\epsilon[F_{2W}(T_W^4 - T_2^4) + (1 - F_{2W})(T_S^4 - T_2^4)]$$

The geometric view factor  $F_{2W} = F_{2W}(\theta)$  is the fraction of radiation leaving a small area on the outer pipe surface that is directly intercepted by the wall; and the remaining fraction,  $1 - F_{2W}$  is absorbed by the remainder of the surroundings. It is described in details in Appendix A.

The temperature distribution in the wall of the tube and the fluid at a particular x-location are determined by solving simultaneously the energy equation in the wall and the fluid and the interface and then the solution marches in the x-direction.

Following assumptions are made to model the problem mathematically:

- (a) The heated wall behaves as a black body radiator.
- (b) The surrounding atmosphere is transparent to thermal radiation.
- (c) Natural convection from the heated wall to the tube surface is neglected, radiation being the principal heat transfer mode.
- (d) The flow is steady, laminar and incompressible.
- (e) Physical properties of both the fluid and the tube wall are constant.

- (f) The tube is smooth so that frictional loss is negligible.
- (g) Axial conduction in the fluid is neglected as in the present case  $RePr$  is greater than 100.
- (h) Axial conduction in the tube wall is neglected as the tube is sufficiently long.
- (i) Viscous dissipation is neglected as it is a low speed flow.

## 2.2 Basic Equations

For incompressible, constant property fluid, the combined momentum and continuity equation for fully developed flow becomes,

$$\frac{1}{r} \frac{d}{dr} (r \frac{du}{dr}) - \frac{1}{\mu} \frac{dp}{dx} = 0$$

with the boundary conditions,

- (i) no slip condition,  $u = 0$  at  $r = r_1$
  - (ii) the velocity is maximum at the centre i.e.  $\frac{du}{dr} = 0$  at  $r = 0$ .
- Solving the momentum equation subject to the above boundary conditions, the velocity profile obtained as follows:

$$u(r) = - \frac{1}{4\mu} \frac{dp}{dx} [r_1^2 - r^2]$$

$\bar{u}$  = Average velocity (axial)

$$= \frac{u_{max}}{2} = - \frac{1}{8\mu} \frac{dp}{dx} r_1^2$$

as,  $u_{max} = u(r=0) = - \frac{1}{4\mu} \frac{dp}{dx} r_1^2$

Inserting  $\bar{u}$  in the expression of  $u(r)$ , the velocity profile gets the form

$$u(r) = 2\bar{u} u^*(r) \quad (2.1)$$

where  $u^*(r) = 1 - \left(\frac{r}{r_1}\right)^2$  (2.2)

Energy equations:

(i) For solid region;  $\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (2.3)$

with boundary conditions,

at  $r = r_2$ ,  $-k_s \frac{\partial T}{\partial r} = -q_2(\theta) =$  radiation heat flux falling on the upper surface

$$\therefore \frac{\partial T}{\partial r} = \frac{1}{k_s} q_2(\theta) = \frac{\sigma \epsilon}{k_s} [F_{2W}(\theta)(T_W^4 - T_2^4) + (1 - F_{2W}(\theta)) (T_S^4 - T_2^4)]$$

$$\therefore \text{at } r = r_2, \frac{\partial T}{\partial r} = \frac{\sigma \epsilon}{k_s} [F_{2W} T_W^4 + (1 - F_{2W}) T_S^4 - T_2^4] \quad (2.3a)$$

where  $F_{2W} = F_{2W}(\theta)$  = configuration factor, determined by Hottel-Cross-string method (see Appendix A)

$$T_2 = T|_{r=r_2} = T(\theta)$$

(ii) Fluid region;

Energy equation;  $u \frac{\partial T}{\partial x} = \alpha_f \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] \quad (2.4)$

with boundary conditions, at  $r = 0, \frac{\partial T}{\partial r} = 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad (2.4a)$   
 at  $x = 0, T = T_e$

At the interface, the equality of temperature and heat flux exists, so that,

$$\text{at } r = r_1, \quad T|_s = T|_f \quad (2.5)$$

$$\text{and } -k_s \frac{\partial T}{\partial r}|_s = -k_f \frac{\partial T}{\partial r}|_f$$

### 2.3 Fluid Bulk Temperature

For the calculation of the convective heat transfer coefficient  $h(x)$  from the wall to the fluid along the axis of the tube ( $x$ -direction), it is necessary to define a characteristic temperature of the fluid commonly known as 'bulk temperature' or 'mixing cup temperature'. The reason is that in a tube flow there is no easily discernible free-stream condition as is present in the flow over a flat plate. For most tube or channel flow heat transfer problems the topic of central interest is the total energy transferred to the fluid in either an elemental length of the tube or over the entire length of the channel. At any  $x$  position, the temperature that is indicative of the total energy of the flow is an integrated mass-energy average temperature over the entire flow area. The numerator of equation (2.6) below, represents the total energy flow through the tube and the denominator of the same equation represents the product of mass flow and specific heat integrated over the flow area. The bulk temperature is thus representative of the total energy of the flow at a particular location. ( $x$ ).

Mathematically the bulk temperature for the fluid is defined as,

$$T_{fb} = \frac{\int_0^{2\pi} \int_0^{r_1} \rho C_p u T r dr d\theta}{\int_0^{2\pi} \int_0^{r_1} \rho C_p u r dr d\theta} \quad (2.6)$$

Thus

$$T_{fb} = T_{fb}(x)$$

The heat flux from the wall of the tube to the fluid at a given  $x$ -location and circumferential location  $\theta$ ,  $q_1(x, \theta)$  is defined as,

$$q_1(x, \theta) = -[-k_s \frac{\partial T}{\partial r}]_{r=r_1, s} = k_s \frac{\partial T}{\partial r} \Big|_{r=r_1, s} \quad (2.7)$$

and the average heat flux at a given  $x$  is defined as,

$$\bar{q}_1(x) = \frac{1}{2\pi} \int_0^{2\pi} q_1 d\theta \quad (2.7a)$$

The average wall temperature of the tube at a given  $x$  is expressed as,

$$\bar{T}_1(x) = \frac{1}{2\pi} \int_0^{2\pi} T_1 d\theta \quad (2.8)$$

where  $T_1 = T_1(x, \theta) = \text{Interface temperature} = T \Big|_{r=r_1}$   $(2.8a)$

Local heat transfer coefficient,  $h_x$  can be found by the following heat flux balance equation;

$$\begin{aligned} \bar{q}_1(x) &= h_x (\bar{T}_1 - T_{fb}) \\ \text{or, } h_x &= \frac{\bar{q}_1(x)}{\bar{T}_1 - T_{fb}} \end{aligned} \quad (2.9)$$

The local Nusselt number,  $Nu_x$  is found by,

$$Nu_x = \frac{h_x(2r_1)}{k_f} \quad (2.10)$$

CHAPTER 3METHOD OF SOLUTION3.1 Finite Difference Approximations

The energy equations of the solid and the fluid along with the compatibility conditions at the interface are discretised using finite difference approximations as described below.

A typical orthogonal mesh system for using the Finite Difference approximation is shown in Figure 2.

The following finite difference approximations are used.

- (i) Using central difference formulae for the terms  $\frac{\partial^2 T}{\partial r^2}$ ,  $\frac{\partial T}{\partial r}$  and  $\frac{\partial^2 T}{\partial \theta^2}$  at the point  $(i, j)$ ,

$$\frac{\partial^2 T}{\partial r^2} = \frac{1}{(\Delta r)^2} [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}]$$

$$\frac{\partial^2 T}{\partial \theta^2} = \frac{1}{(\Delta \theta)^2} [T_{i,j+1} - 2T_{i,j} + T_{i,j-1}]$$

$$\frac{\partial T}{\partial r} = \frac{1}{2(\Delta r)} [T_{i+1,j} - T_{i-1,j}]$$

where  $\Delta r$  = uniform step size in  $r$ -direction

$\Delta \theta$  = uniform step size in  $\theta$ -direction

$i$  = index in  $r$ -direction

$j$  = index in  $\theta$ -direction.

- (ii) Using backward difference formula for the convection term

$\frac{\partial T}{\partial x}$  at a  $x$ -station,  $k+1$ ,

$$\frac{\partial T}{\partial x} = \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta x}$$

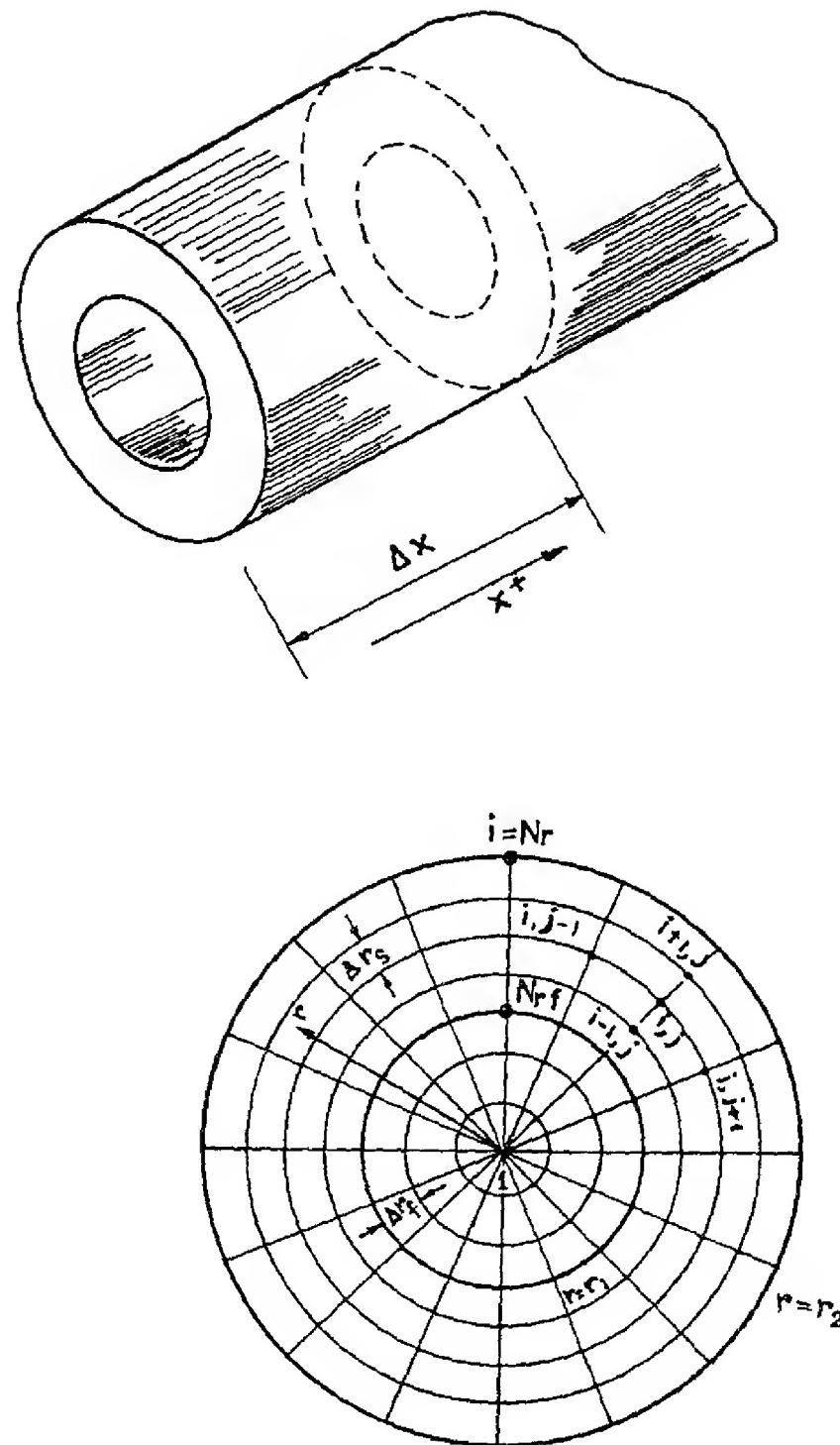


FIG. 2 A TYPICAL FINITE DIFFERENCE GRID SYSTEM FOR THE PROBLEM

where  $T_{i,j,k}$  = Temperature of the point (i, j) at the previous x-station.

Also  $\frac{\partial T}{\partial r}$  term can be discretised by backward difference formula as,

$$\left. \frac{\partial T}{\partial r} \right|_{i+1,j} = \frac{T_{i+1,j} - T_{i,j}}{\Delta r}$$

### 3.1.1 Discretization of the Energy Equation for Solid

The equation (2.3) subjected to the boundary condition (2.3a) can be discretized as follows,

$$\text{Equation (2.3); } \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} = 0$$

Boundary condition (2.3a); at  $r = r_2$ ;

$$\frac{\partial T}{\partial r} = \frac{\sigma \epsilon}{k_s} [F_{2W} T_W^4 + (1 - F_{2W}) T_S^4 - T_2^4]$$

Refer to Figure 2.

Let,  $N_r$  = Number of grid points in r-direction for the entire fluid-solid region

$N_\phi$  = Number of grid points in  $\phi$ -direction, both for solid and fluid region

$N_{rf}$  = Number of grid points in r-direction, only in the fluid region.

As x-direction is taken infinitely long, no specific number of grid points are taken for this direction. By marching procedure the solution can be continued upto any length and it can be terminated as per requirement.

The uniform step sizes are,

$$\Delta r_s = \frac{(r_2 - r_1)}{(N_r - N_{rf})}$$

$$\Delta\phi = \frac{2\pi}{N_\phi}$$

and  $\Delta r_f = \frac{r_1}{(N_{rf} - 1)}$

Discretizing equation (2.3) by finite difference approximation, as

$$\begin{aligned} \frac{1}{(\Delta r_s)^2} [T_{i-1,j} - 2T_{i,j} + T_{i+1,j}] + \frac{1}{r_i} \frac{1}{2\Delta r_s} [T_{i+1,j} - T_{i-1,j}] \\ + \frac{1}{r_i^2} \frac{1}{(\Delta\phi)^2} [T_{i,j-1} - 2T_{i,j} + T_{i,j+1}] = 0 \end{aligned}$$

Simplifying the above equation,

$$\begin{aligned} (2r_i^2 - r_i \Delta r_s)T_{i-1,j} - [4r_i^2 + 4(\frac{\Delta r_s}{\Delta\phi})^2]T_{i,j} \\ + (2r_i^2 + r_i \Delta r_s)T_{i+1,j} = -2(\frac{\Delta r_s}{\Delta\phi})^2 [T_{i,j-1} + T_{i,j+1}] \end{aligned}$$

Writing the above equation in the standard tridiagonal matrix form,

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.1)$$

where  $A_i = 2r_i^2 - r_i \Delta r_s$

$$B_i = -[4r_i^2 + 4(\frac{\Delta r_s}{\Delta\phi})^2]$$

$$C_i = 2r_i^2 + r_i \Delta r_s$$

$$D_i = -2(\frac{\Delta r_s}{\Delta\phi})^2 [T_{i,j-1} + T_{i,j+1}]$$

The above equation is valid for any interior point  $(i, j)$  of the tube wall except the boundary points, i.e. for all nodes numbering from  $i = N_{rf} + 1$  to  $i = N_r - 1$  and in  $\phi$ -direction all points except the points at  $\phi = 0$  and  $\phi = (2\pi - \phi)$ . i.e.  $j = 1$  and  $j = N_\phi$ .

More correctly saying, the equation (3.1) is valid also for  $j = 1$  and  $j = N_\phi$ , but for computational purpose the equation is slightly modified for all points with  $j = 1$  and  $j = N_\phi$ , as described below.

For  $j = 1$ , the point  $(j-1)$  physically represents the point ' $N_\phi$ '. Therefore the equation (3.1) is modified by replacing  $(j-1)$  by  $N_\phi$  for  $j = 1$  and  $(j+1)$  by 1 for  $j = N_\phi$ , both changes occur in the expression of  $D_i$ .

Rewriting equation (3.1) for different  $j$ 's the general form becomes,

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.1)$$

with  $A_i = 2r_i^2 - r_i \Delta r_s$

$$B_i = - [4 r_i^2 + 4(\frac{\Delta r_s}{\Delta \phi})^2] \quad , \text{ for all } j$$

$$C_i = 2r_i^2 + r_i \Delta r_s$$

and  $D_i = - 2(\frac{\Delta r_s}{\Delta \phi})^2 [T_{i,j-1} + T_{i,j+1}] \quad \text{for } 2 \leq j \leq (N_\phi - 1)$

$$= - 2(\frac{\Delta r_s}{\Delta \phi})^2 [T_{i,N_\phi} + T_{i,2}] \quad \text{for } j = 1$$

$$= - 2(\frac{\Delta r_s}{\Delta \phi})^2 [T_{i,1} + T_{i,N_\phi-1}] \quad \text{for } j = N_\phi$$

However the equation (3.1) is only valid for interior points of the tube wall.

To get the discretised energy equation for the upper surface, i.e.  $i = N_r$  ( $r = r_2$ ), the boundary condition (2.3a) is used to modify equation (3.1) as described below.

Boundary condition (2.3a) is, at  $r = r_2$  (i.e.  $i = N_r$ ),

$$\frac{\partial T}{\partial r} = \frac{\sigma\varepsilon}{k_s} [F_{2W} T_W^4 + (1 - F_{2W}) T_S^4 - T_2^4]$$

where  $T_2$  = temperature of the tube at  $r = r_2$ , this is represented by  $T$ .

Putting  $T$  in place of  $T_2$  and discretising equation (2.3a)

$$\frac{T_{i+1,j} - T_{i-1,j}}{2\Delta r_s} = \frac{\sigma\varepsilon}{k_s} [F_{2W_j} T_W^4 + (1 - F_{2W_j}) T_S^4 - T_{i,j}^4]$$

but as  $T_{i,j}^4$  term introduces non-linearity in the discretized equation it is necessary to linearize this term using Patankar's approach [10] to linearization of the source term.

According to this approach, the non-linear term should be linearized in such a manner so that fast convergence is attained. It also suggests that when the coefficient of the transformed linear term is equal to the slope of the original nonlinear term the convergence is fastest, however this is not true always.

Using this approach  $T_{i,j}^4$  term is linearized as,

$$T_{i,j}^4 = 4(T_{i,j}^n)^3 T_{i,j} - 3(T_{i,j}^n)^4$$

where  $T_{i,j}^n$  = Temperature at point  $(i, j)$  in the previous iteration.

Putting this in the discretized equation of the boundary condition (2.3a), the equation becomes,

$$\frac{T_{i+1,j} - T_{i-1,j}}{2\Delta r_s} = \frac{\sigma\varepsilon}{k_s} [F_{2W_j} T_W^4 + (1 - F_{2W_j}) T_S^4 + 3(T_{i,j}^n)^4 - 4(T_{i,j}^n)^3 T_{i,j}]$$

Simplifying it,

$$T_{i+1,j} = \frac{2\sigma\varepsilon}{k_s} \Delta r_s [F_{2W_j} T_W^4 + (1 - F_{2W_j}) T_S^4 + 3(T_{i,j}^n)^4 - 4(T_{i,j}^n)^3 T_{i,j}] + T_{i-1,j}$$

Putting this expression in equation (3.1) the discretized energy equation for  $i = N_r$  is obtained as follows.

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.1a)$$

$$\text{where } A_i = 4r_i^2 = 4r_2^2 \quad (\text{as } i = N_r \Rightarrow r_i = r_2)$$

$$B_i = -4S_2(T_{i,j}^n)^3 - 4r_2^2 - 4(\frac{\Delta r_s}{\Delta \theta})^2$$

$$C_i = 0$$

$$D_i = -2(\frac{\Delta r_s}{\Delta \theta})^2 [T_{i,j-1} + T_{i,j+1}] - S_2 [F_{2W_j} T_W^4 + (1 - F_{2W_j}) T_S^4 + 3(T_{i,j}^n)^4]$$

$$\text{where } S_2 = \frac{2\sigma\varepsilon}{k_s} \Delta r_s (2r_2^2 + r_2 \Delta r_s).$$

The above equation is valid for  $i = N_r$  only, here also  $D_i$  is modified for  $j = 1$  and  $j = N_\theta$  as before, i.e. replacing  $(j-1)$  by  $N_\theta$  for  $j = 1$  and  $(j+1)$  by 1 for  $j = N_\theta$ .

### 3.1.2 Discretization of the Energy Equation for Fluid

The energy equation for fluid (Equation (2.4)) is changed by inserting the expression of  $u(r)$  into it as below.

$$2\bar{u} u^*(r) \frac{\partial T}{\partial x} = \alpha_f \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right]$$

or,

$$\frac{2\bar{u}}{\alpha_f} u^*(r) \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}$$

Discretizing this equation using finite difference approximation for any point  $(i, j)$  in the fluid region and at a  $x$ -station  $(k+1)$ ,

$$\begin{aligned} \frac{2\bar{u} u_i^*}{\alpha_f} \cdot \frac{T_{i,j,k+1} - T_{i,j,k}}{\Delta x} &= \frac{1}{(\Delta r_f)^2} [T_{i-1,j} - 2T_{i,j} + T_{i,j+1}]_{k+1} \\ &+ \frac{1}{r_i} \frac{1}{2\Delta r_f} [T_{i+1,j} - T_{i-1,j}]_{k+1} + \frac{1}{r_i^2} \frac{1}{(\Delta \theta)^2} [T_{i,j-1} \\ &- 2T_{i,j} + T_{i,j+1}]_{k+1} \end{aligned}$$

For simplification the two dimensional notation for  $T$  is used, replacing  $T_{i,j,k}$  by  $T_{P_{i,j}}$  and  $T_{i,j,k+1}$  by  $T_{i,j}$ .

Simplifying the above discretized energy equation for fluid and writing it in the standard form of tri-diagonal matrix, as below,

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.2a)$$

$$\text{where } A_i = 2r_i^2 - \Delta r_f r_i$$

$$B_i = -[4r_i^2 + 4\left(\frac{\Delta r_f}{\Delta \theta}\right)^2 + C r_i^2 u_i^*]$$

$$C_i = 2r_i^2 + \Delta r_f r_i$$

$$D_i = -2\left(\frac{\Delta r_f}{\Delta \theta}\right)^2 [T_{i,j-1} + T_{i,j+1}] - C u_i^* r_i^2 T_{P_{i,j}}$$

where  $C = \frac{4\bar{u}(\Delta r_f)^2}{\alpha_f \Delta x}$

Equation (3.2a) is valid for  $i = 2$  to  $i = N_{rf} - 1$ . As in the case of solid region, this equation is also modified for  $j = 1$  and  $j = N_\theta$  and the modification takes place in the expression of  $D_i$ .

The boundary condition (2.4a) is discretized as below.

For  $i = 1$ , i.e.  $r = 0$ ;  $\frac{\partial T}{\partial r} = 0$

discretizing it;  $\frac{T_{2,j} - T_{1,j}}{\Delta r_f} = 0$

or,  $-T_{1,j} + T_{2,j} = 0 \quad (3.2b)$

Combining equations (3.2a) and (3.2b) the general form of the discretized energy equation for fluid is obtained as below,

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.2)$$

where  $A_i = 2r_i^2 - r_i \Delta r_f$

$$B_i = -[4r_i^2 + 4\left(\frac{\Delta r_f}{\Delta \theta}\right)^2 + C r_i^2 u_i^*]$$

$$C_i = 2r_i^2 + r_i \Delta r_f$$

$$D_i = -2\left(\frac{\Delta r_f}{\Delta \theta}\right)^2 [T_{i,j-1} + T_{i,j+1}] - C u_i^* r_i^2 T_{P_{i,j}}$$

valid for  $i = 2$  to  $i = N_{rf} - 1$

and also for  $i = 1$ ,  $A_i = 0$

$$B_i = -1$$

$$C_i = 1$$

$$D_i = 0$$

Equation (3.2) is modified for  $j = 1$  and  $j = N_f$ .

### 3.1.3 Discretization of the Compatibility Conditions at the Fluid-Solid Interface

The compatibility conditions (equation (2.5)) are,

$$\text{at } r = r_1, \quad k_s \frac{\partial T}{\partial r} \Big|_s = k_f \frac{\partial T}{\partial r} \Big|_f \quad \text{and} \quad T|_s = T|_f$$

Energy equation for solid, equation (2.3)

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0$$

and energy equation for fluid, equation (2.4)

$$\frac{2\bar{u}}{\alpha_f} u^*(r) \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2}$$

Refer to Figure 3.

From Taylor's series expansion for point  $(i-1, j)$ ,

$$T_{i-1,j} = T_{i,j} - \Delta r_f \left( \frac{\partial T}{\partial r} \right)_{i,j,f} + \frac{\Delta r_f^2}{2} \left( \frac{\partial^2 T}{\partial r^2} \right)_{i,j,f}$$

Solving  $\left( \frac{\partial^2 T}{\partial r^2} \right)_{i,j,f}$  from the above equation

$$\left( \frac{\partial^2 T}{\partial r^2} \right)_{i,j,f} = \frac{2}{(\Delta r_f)^2} [T_{i-1,j} - T_{i,j} + \Delta r_f \left( \frac{\partial T}{\partial r} \right)_{i,j,f}]$$

At the interface  $u_i^* = 0$ , also  $r_i = r_1$ .

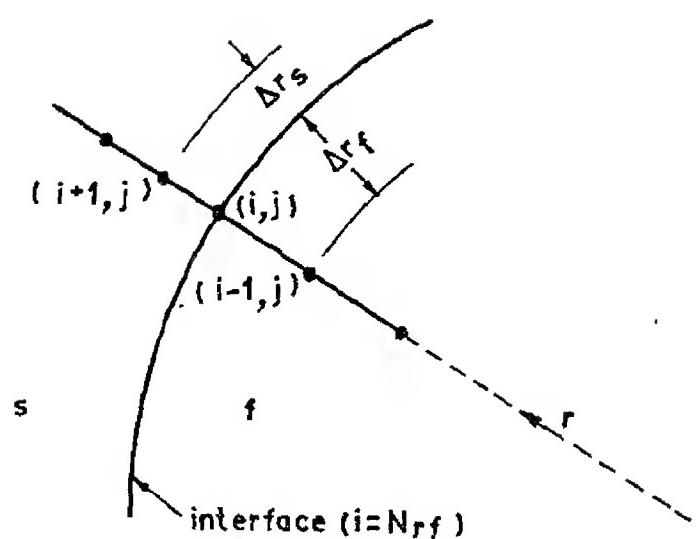


FIG. 3 FINITE DIFFERENCE NODES NEAR THE INTERFACE

Putting all these in the energy equation for fluid,  
i.e. in equation (2.4),

$$0 = \frac{2}{(\Delta r_f)^2} [T_{i-1,j} - T_{i,j} + \Delta r_f (\frac{\partial T}{\partial r})_{i,j,f}] \\ + \frac{1}{r_1} (\frac{\partial T}{\partial r})_{i,j,f} + \frac{1}{r_1^2} (\frac{\partial^2 T}{\partial \theta^2})_{i,j,f}$$

Simplifying it,

$$(\frac{\partial T}{\partial r})_{i,j,f} = \frac{2r_1}{2r_1 \Delta r_f + (\Delta r_f)^2} [T_{i,j} - T_{i-1,j} - \frac{(\Delta r_f)^2}{2r_1^2} (\frac{\partial^2 T}{\partial \theta^2})_{i,j,f}] \quad (3.3a)$$

Expressing  $T_{i+1,j}$  in Taylor's series as,

$$T_{i+1,j} = T_{i,j} + \Delta r_s (\frac{\partial T}{\partial r})_{i,j,s} + \frac{(\Delta r_s)^2}{2} (\frac{\partial^2 T}{\partial r^2})_{i,j,s}$$

thereby,

$$(\frac{\partial^2 T}{\partial r^2})_{i,j,s} = \frac{2}{(\Delta r_s)^2} [T_{i+1,j} - T_{i,j} - r_s (\frac{\partial T}{\partial r})_{i,j,s}]$$

Using this in the energy equation of solid and simplifying it,

the  $(\frac{\partial T}{\partial r})_{i,j,s}$  term is expressed as below,

$$(\frac{\partial T}{\partial r})_{i,j,s} = \frac{2r_1}{2r_1 \Delta r_s - (\Delta r_s)^2} [T_{i+1,j} - T_{i,j} - \frac{(\Delta r_s)^2}{2r_1^2} (\frac{\partial^2 T}{\partial \theta^2})_{i,j,s}] \quad (3.3b)$$

Ultimately two expressions are obtained for the temperature gradient at the interface for fluid and solid regions as,

$$(\frac{\partial T}{\partial r})_{i,j,f} = -\frac{2r_1}{\Delta r_f (2r_1 + \Delta r_f)} [T_{i-1,j} - T_{i,j} + \frac{(\Delta r_f)^2}{2r_1^2} (\frac{\partial^2 T}{\partial \theta^2})_{i,j,f}]$$

and

$$\left(\frac{\partial T}{\partial r}\right)_{i,j,s} = \frac{2r_1}{\Delta r_s(2r_1 - \Delta r_s)} [T_{i+1,j} - T_{i,j} + \frac{(\Delta r_s)^2}{2r_1^2} \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,s}]$$

The compatibility conditions (equation (2.5)), i.e.  $T|_s = T|_f$  at  $r = r_1$  and

$$k_s \frac{\partial T}{\partial r}|_s = k_f \frac{\partial T}{\partial r}|_f \quad \text{at } r = r_1$$

$\left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,f}$  and  $\left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,s}$  will be equal and both of them have same expression because the temperature of fluid and solid are the same at the interface and  $\frac{\partial^2 T}{\partial \phi^2}$  is expressed as,

$$\begin{aligned} \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,s} &= \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,f} = \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j,r=r_1} \\ &= \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{(\Delta \phi)^2} \end{aligned}$$

Using the heat flux equality condition, i.e.

$$k_s \left(\frac{\partial T}{\partial r}\right)_{i,j,s} = k_f \left(\frac{\partial T}{\partial r}\right)_{i,j,f}$$

and expressing  $\left(\frac{\partial T}{\partial r}\right)_{i,j,s}$  and  $\left(\frac{\partial T}{\partial r}\right)_{i,j,f}$  as in equations (3.3a) and (3.3b), the following equation is resulted

$$\begin{aligned} \frac{k_s}{k_f} \cdot \frac{2r_1}{2r_1 \Delta r_s - (\Delta r_s)^2} [T_{i+1,j} - T_{i,j} + \frac{(\Delta r_s)^2}{2r_1^2} \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j}] \\ = \frac{2r_1}{2r_1 \Delta r_f + (\Delta r_f)^2} [T_{i,j} - T_{i-1,j} - \frac{(\Delta r_f)^2}{2r_1^2} \left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j}] \end{aligned}$$

Using the expression of  $\left(\frac{\partial^2 T}{\partial \phi^2}\right)_{i,j}$  in the above equation and after necessary simplifications the standard discretized equation for the interface is obtained as below,

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (\text{for } i = N_{rf}) \quad (3.3)$$

where,  $A_i = 2r_1^2 = C_i$

$$B_i = - (2r_1^2 R + 2r_1^2 + RR)$$

$$D_i = - RR(T_{i,j+1} + T_{i,j-1})$$

where  $R = \frac{k_s}{k_f} \cdot \frac{2r_1 \Delta r_f + (\Delta r_f)^2}{2r_1 \Delta r_s - (\Delta r_s)^2}$

and  $RR = \frac{R(\Delta r_s)^2 + (\Delta r_f)^2}{(\Delta \phi)^2}$

Equation (3.3) is valid for all j's except  $j = 1$  and  $j = N_\phi$ .

For these two j-stations the equation is modified by replacing  $(j-1)$  by  $N_\phi$  for  $j = 1$  and  $(j+1)$  by 1 for  $j = N_\phi$ .

### 3.1.4 Simplified Equations for $T_{fb}$ and $q_1(x, \phi)$

From equation (2.6),  $T_{fb} = \frac{\int_0^{2\pi} \int_0^{r_1} u T r dr d\phi}{\int_0^{2\pi} \int_0^{r_1} u r dr d\phi}$ , as

$\rho C_p$  = Constant for the fluid.

Putting the expression of  $u(r) = 2\bar{u} \cdot u^*(r)$ , where  $u^*(r) = 1 - \left(\frac{r}{r_1}\right)^2$ , then simplifying the above expression for  $T_{fb}$  the following expression is obtained,

$$T_{fb} = \frac{2}{\pi r_1^2} \int_0^{2\pi} \int_0^{r_1} u^*(r) T r dr d\phi \quad (3.4)$$

The heat flux,  $q_1(x, \phi)$ ,

$$q_1(x, \theta) = k_s \frac{\partial T}{\partial r} \Big|_{r=r_1, s} = k_s \frac{T_{N_{rf}+1, j} - T_{N_{rf}, j}}{\Delta r_s}$$
(3.5)

### 3.2 Numerical Solution of the Discretized Equations

The set of discretized equations to be solved are listed below.

$$\text{Equation (3.2), } A_i T_{i-1, j} + B_i T_{i, j} + C_i T_{i+1, j} = D_i$$

$$\text{where } A_i = 2r_i^2 - r_i \Delta r_f$$

$$B_i = - [4r_i^2 + 4(\frac{\Delta r_f}{\Delta \theta})^2 + C r_i^2 u_i^*]$$

$$C_i = 2r_i^2 + r_i \Delta r_f$$

$$D_i = - 2(\frac{\Delta r_f}{\Delta \theta})^2 [T_{i, j-1} + T_{i, j+1}] - C u_i^* r_i^2 T_{P_i, j}$$

for  $i = 2$  to  $i = N_{rf} - 1$

and

$$A_i = 0$$

$$B_i = -1, \quad \text{for } i = 1$$

$$C_i = 1$$

$$D_i = 0$$

$$\text{Equation (3.3), } A_i T_{i-1, j} + B_i T_{i, j} + C_i T_{i+1, j} = D_i$$

$$\text{where } A_i = 2r_1^2 = C_i$$

$$B_i = - [2r_1^2 R + 2r_1^2 + RR]$$

$$D_i = -RR(T_{i,j+1} + T_{i,j-1})$$

$$R = \frac{k_s}{k_f} \cdot \frac{2r_1 \Delta r_f + (\Delta r_f)^2}{2r_1 \Delta r_s - (\Delta r_s)^2}$$

and  $RR = \frac{R(\Delta r_s)^2 + (\Delta r_f)^2}{(\Delta \phi)^2}$

Equations (3.1) and (3.1a) combinedly written as

$$A_i T_{i-1,j} + B_i T_{i,j} + C_i T_{i+1,j} = D_i \quad (3.1b)$$

where  $A_i = 2r_i^2 - r_i \Delta r_s$

$$B_i = -[4r_i^2 + 4(\frac{\Delta r_s}{\Delta \phi})^2] \quad \text{for } i = N_{rf} + 1$$

$$C_i = 2r_i^2 + r_i \Delta r_s \quad \text{to } i = N_r - 1$$

$$D_i = -2(\frac{\Delta r_s}{\Delta \phi})^2 [T_{i,j-1} + T_{i,j+1}]$$

and  $A_i = 3r_2^2$

$$B_i = -[4S_2(T_{i,j}^n)^3 + 4r_2^2 + 4(\frac{\Delta r_s}{\Delta \phi})^2]$$

$$C_i = 0$$

$$D_i = -2(\frac{\Delta r_s}{\Delta \phi})^2 [T_{i,j-1} + T_{i,j+1}]$$

$$- S_2 [F_{2W_j} T_W^4 + (1 - F_{2W_j}) T_S^4 + 3(T_{i,j}^n)^4]$$

where  $S_2 = \frac{2\sigma\varepsilon}{k_s} \Delta r_s (2r_2^2 + r_2 \Delta r_s)$ , valid for  $i = N_r$ .

All the above equations are to be modified for  $j = 1$  and  $j = N_\phi$ , the modifications will be only in the expression of  $D_i$ . For

$j = 1$ ,  $(j - 1)$  is to be replaced by  $N_f$  and for  $j = N_f$ ,  $(j + 1)$  is substituted by 1.

$$\text{Equation (3.4), } T_{fb}(x) = \frac{2}{\pi r_1^2} \int_0^{2\pi} \int_0^{r_1} u^*(r) T r dr d\theta$$

$$\text{Equation (3.5), } q_1(x, \theta) = k_s \frac{T_{N_{rf}+1,j} - T_{N_{rf},j}}{r_s}$$

$$\text{Equation (2.7a), } \bar{q}_1(x) = \frac{1}{2\pi} \int_0^{2\pi} q_1 d\theta$$

$$\text{Equation (2.8), } \bar{T}_1(x) = \frac{1}{2\pi} \int_0^{2\pi} T_1 d\theta$$

$$\text{Equation (2.9), } h_x = \frac{\bar{q}_1(x)}{\bar{T}_1 - T_{fb}}$$

$$\text{Equation (2.10), } Nu_x = \frac{h_x (2r_1)}{k_f}$$

The set of discretized equations (3.2), (3.3) and (3.1b) for fluid, interface and solid are to be solved by a suitable numerical technique.

In the present analysis a line-by-line method, suggested by Patankar [10], is used. The line-by-line method helps to solve the set of simultaneous linear algebraic equations by Tri-Diagonal Matrix Algorithm (TDMA) method.

The line-by-line method is described in Appendix-B in great detail.

### 3.3 Algorithm for the Numerical Solution

To solve the discretized equation (3.2), (3.3) and (3.1b) to get the temperature at a particular x-location, the following steps are taken.

- (a) The computations are started at the location  $x = \Delta x$  using the known inlet temperature at the entrance of the tube.
- (b) The r-direction lines are selected to initiate the solution procedure by line-by-line method.
- (c) All the temperatures are assumed initially, both in the fluid and solid region.
- (d) Start solving by TDMA with the line  $\emptyset = 0$ .  
The temperature at all the grids of this line are solved.
- (e) Go to the next line along  $\emptyset$ -direction (i.e. at  $\emptyset = \Delta \emptyset$ ) and solve the discretized equations to get temperature at all nodes on that line.
- (f) This procedure is followed one after another for all the r-direction lines.
- (g) The steps 'd' to 'f' are repeated till the process converges, i.e. the temperature at any node does not change further with iterative procedure.

After getting the converged values of temperature, they are stored for that particular x-station. Then the fluid bulk temperature, the average heat flux, the local values of convective heat transfer coefficient and the Nusselt number are calculated following the equations (3.4), (3.5) and (2.7a)

to (2.10). The integrations involved are done by Simpson's rule of numerical integration.

The procedure then marches in x-direction and continued till the fully developed situation is reached. The fully developed situation is indicated when there is no considerable change in  $Nu_x$  along x.

The accompanying Flow-chart (Fig. 4) represents the algorithm pictorially. The input data to the program and the listing of the computer program are given in Appendix-C and Appendix-D respectively.

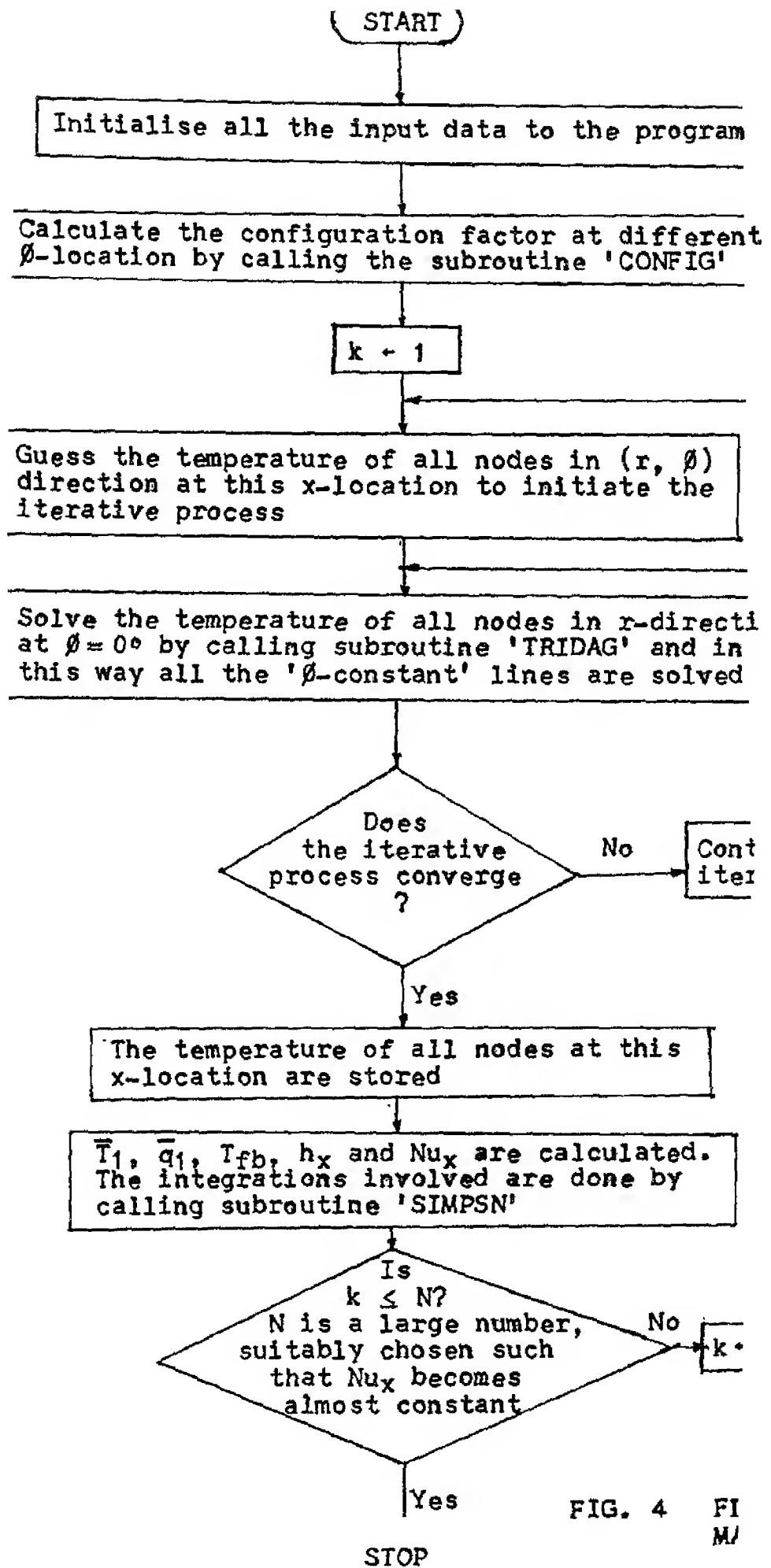


FIG. 4 FI  
MJ

## CHAPTER 4

### RESULTS AND DISCUSSIONS

Figure 5 shows the temperature distribution in the tube wall for different conductivities of the tube material at the circumferential location,  $\theta = 0^\circ$  (highest point on the tube). The temperatures in the tube wall and also the temperature gradient are higher for low conductivity material than for high conductivity material. This is because for low conductivity material the resistance to heat transfer is high and hence higher temperature-gradient. Also, because of less heat transfer to the fluid, the inner wall is cooled less by the flowing fluid which results in raising the temperature of the wall. This suggests that in case of low conductivity tube material there is a chance of burn out of the tube at the point of highest temperature which in turn results in leakage of the fluid.

In Figure 6, the effect of wall thickness on the temperature distribution in the tube wall is depicted. It is observed that with the decrease of tube thickness, the temperature gradient decreases and the temperature level in the tube increases. So, a thin tube with low conductivity is not desirable as it may give rise to hot spots occurring due to high temperatures on the surface of the tube. The nature of the curves satisfy the physics as with the reduction of thickness, the thermal resistance to heat flow decreases and hence

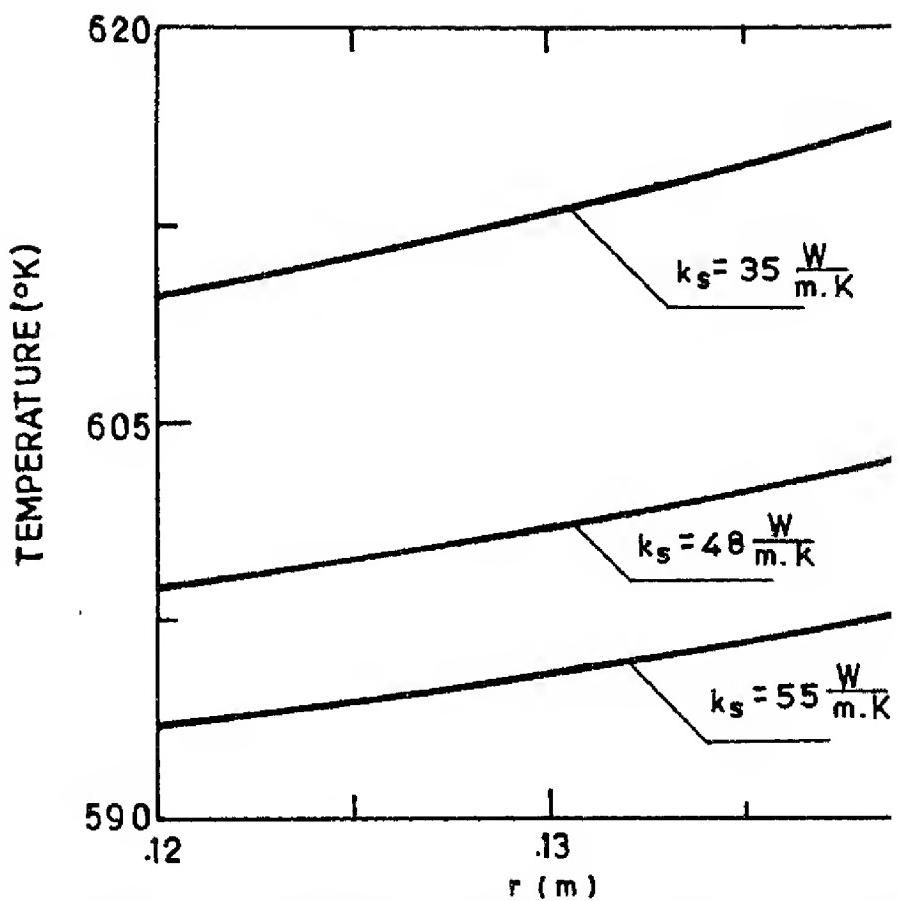


FIG. 5 RADIAL VARIATION OF THE TEMPERATURE  
TUBE WALL FOR VARIOUS CONDUCTIVITIES  
TUBE MATERIAL AT  $\phi = 0^{\circ}$   
( $d = 0.1$ ,  $Th = 0.02$ ,  $W = 1.0$ ,  $Pr = 51$ ,  $Re = 2000$ )

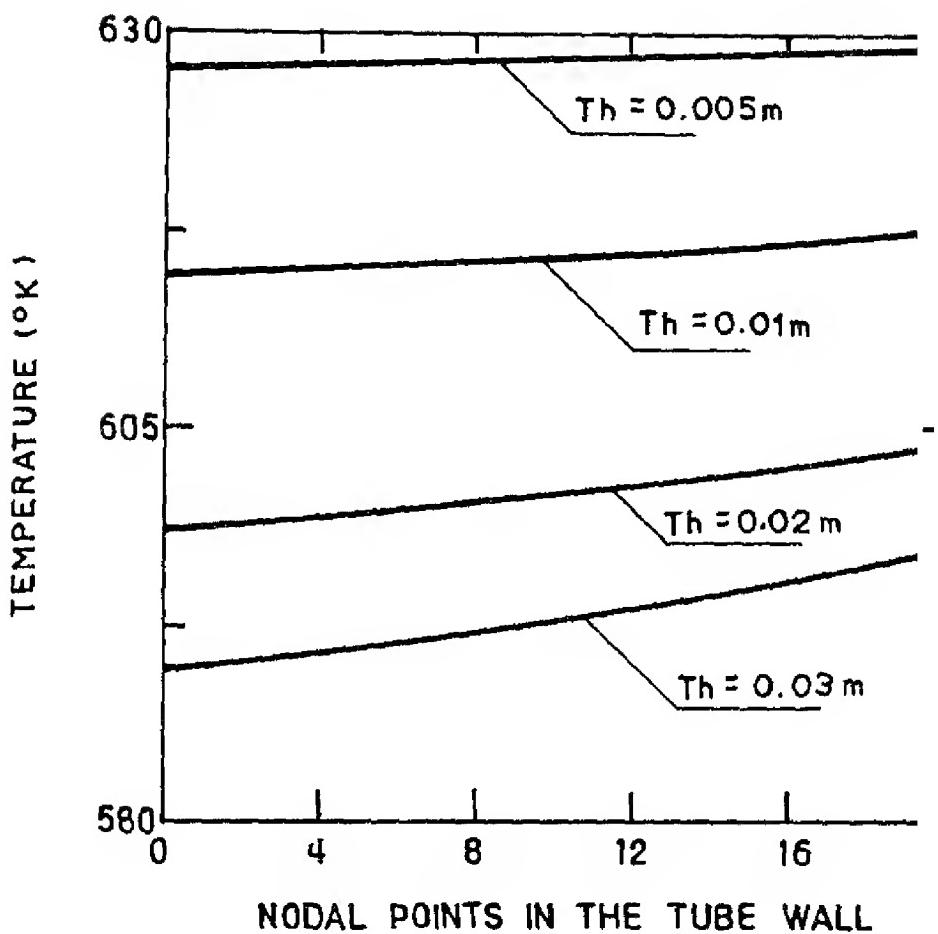


FIG.6 TEMPERATURE VARIATION IN THE TUBE FOR DIFFERENT THICKNESS OF THE  
 $\phi = 0^{\circ}$  ( $d = 0.1$ ,  $W = 1.0$ ,  $k_s = 48$ ,  $\text{Pr} = 5$ )

decreases in temperature gradient. Also most of the radiation heat input is used to raise the temperature.

Figure 7 shows the effect of vertical location ( $d$ ) of the tube with respect to the heated wall on the temperature distribution in the tube wall. As expected, the temperature level shoots up when the tube is placed closer to the wall as it receives more heat from the wall. However, the temperature gradient remains independent of the location of the tube ( $d$ ) as the tube thickness and thermal conductivity are same for all these cases.

Figure 8 shows the circumferential inner tube wall temperature distribution (fluid-solid interface temperature) at a particular  $x$ -location for different widths ( $W$ ) of the heated wall when the tube is placed equidistant from the two edges of the wall. As one would expect, the curves show symmetric nature about  $\theta = 180^\circ$  as the radiation heat flux between  $0^\circ$  and  $180^\circ$  is same as that between  $180^\circ$  and  $360^\circ$ . However, the curve for bigger width of the wall is placed above that for smaller widths of the wall. Also it is more flattened. This is quite expected, as the tube receives more radiation when the heated wall is more wide. The flattened nature is because the circumferential radiation heat flux is more uniform in nature in case of larger width ( $W$ ) of the heated wall.

Figure 9 indicates the plots of mean inner wall temperature against the axial distance ( $x$ ) for different Prandtl number fluids, for a constant Reynolds number. It is observed that the mean wall temperature is lower for higher

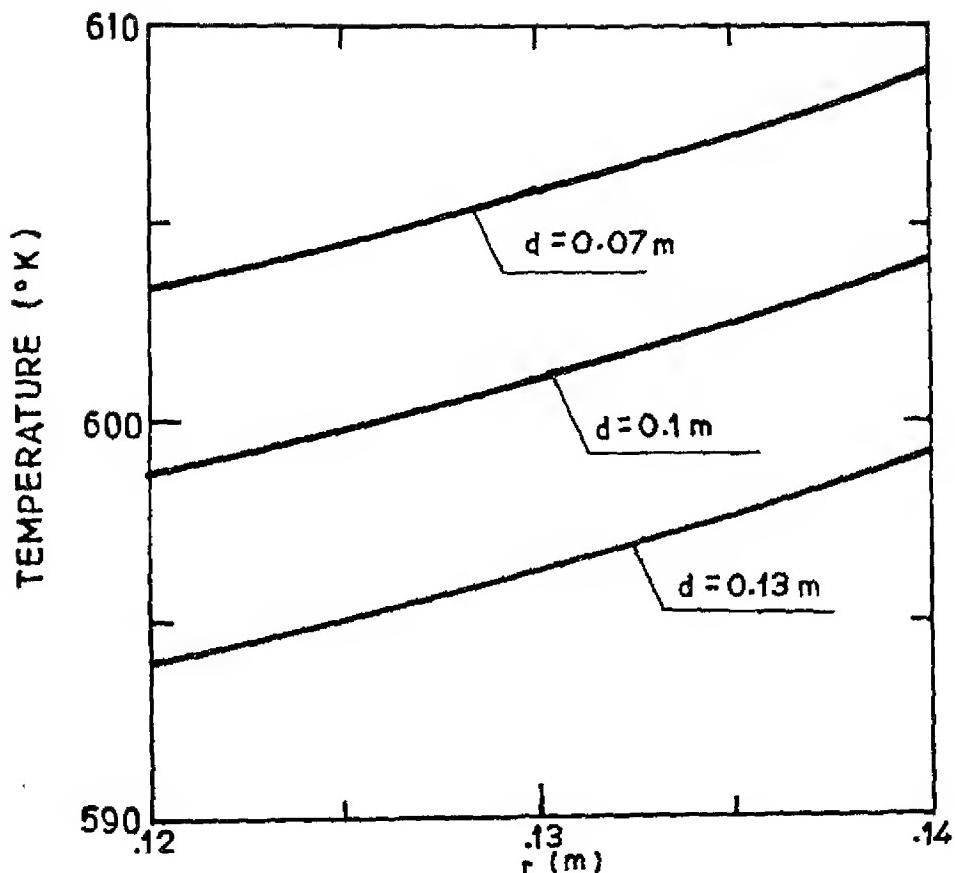
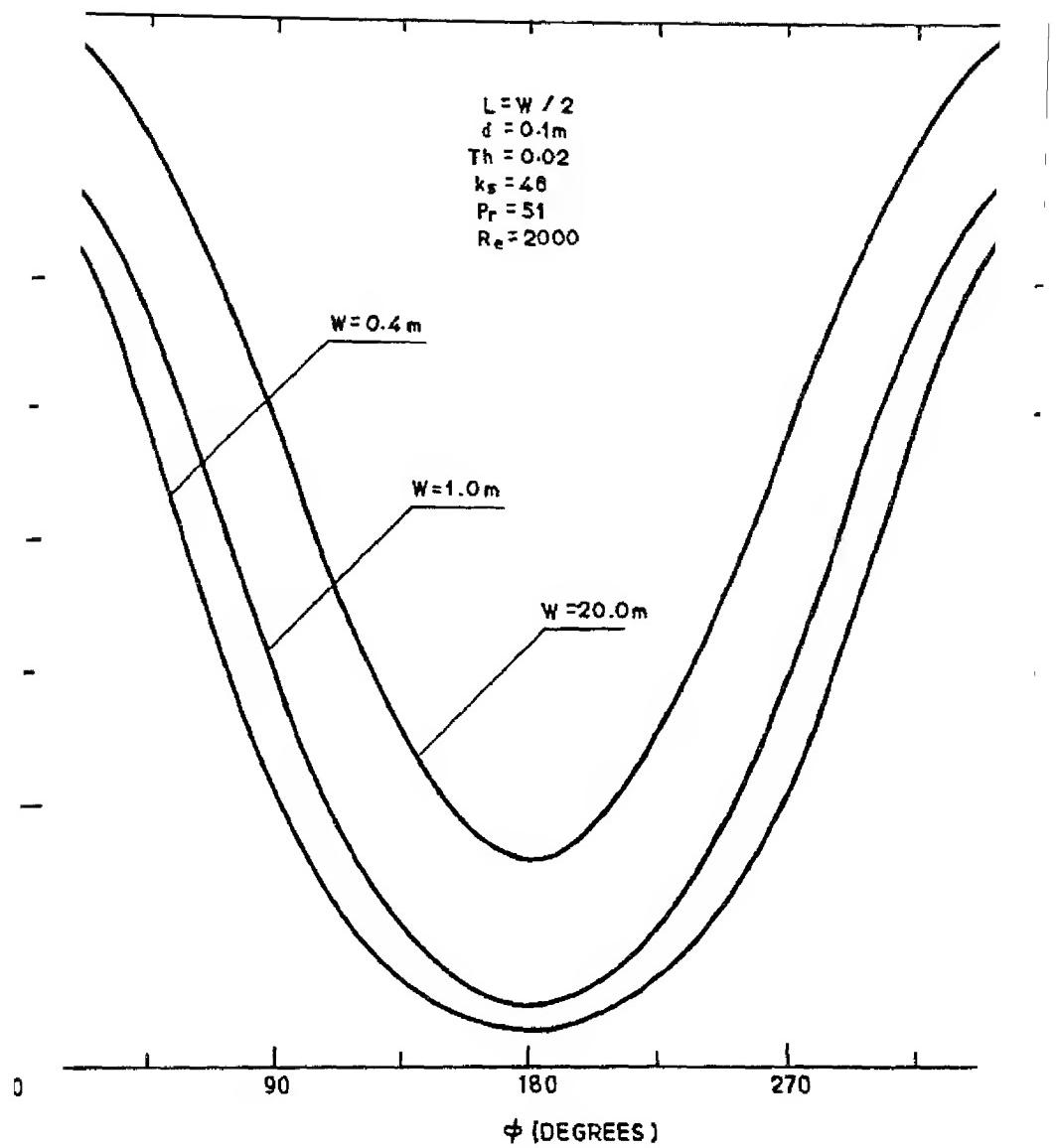


FIG. 7 RADIAL TEMPERATURE DISTRIBUTION IN THE TUBE WALL FOR DIFFERENT VERTICAL POSITION OF THE TUBE ( $d$ ) WITH RESPECT TO THE HEATED WALL ( $\text{Th} = 0.02$ ,  $W = 1.0$ ,  $k_s = 48$ ,  $\text{Pr} = 51$ ,  $\text{Re} = 2000$  )



8 CIRCUMFERENTIAL VARIATION OF THE FLUID-SOLID INTERFACE TEMPERATURE FOR DIFFERENT WIDTHS OF THE HEATED WALL , AT  $x = 0.1r$

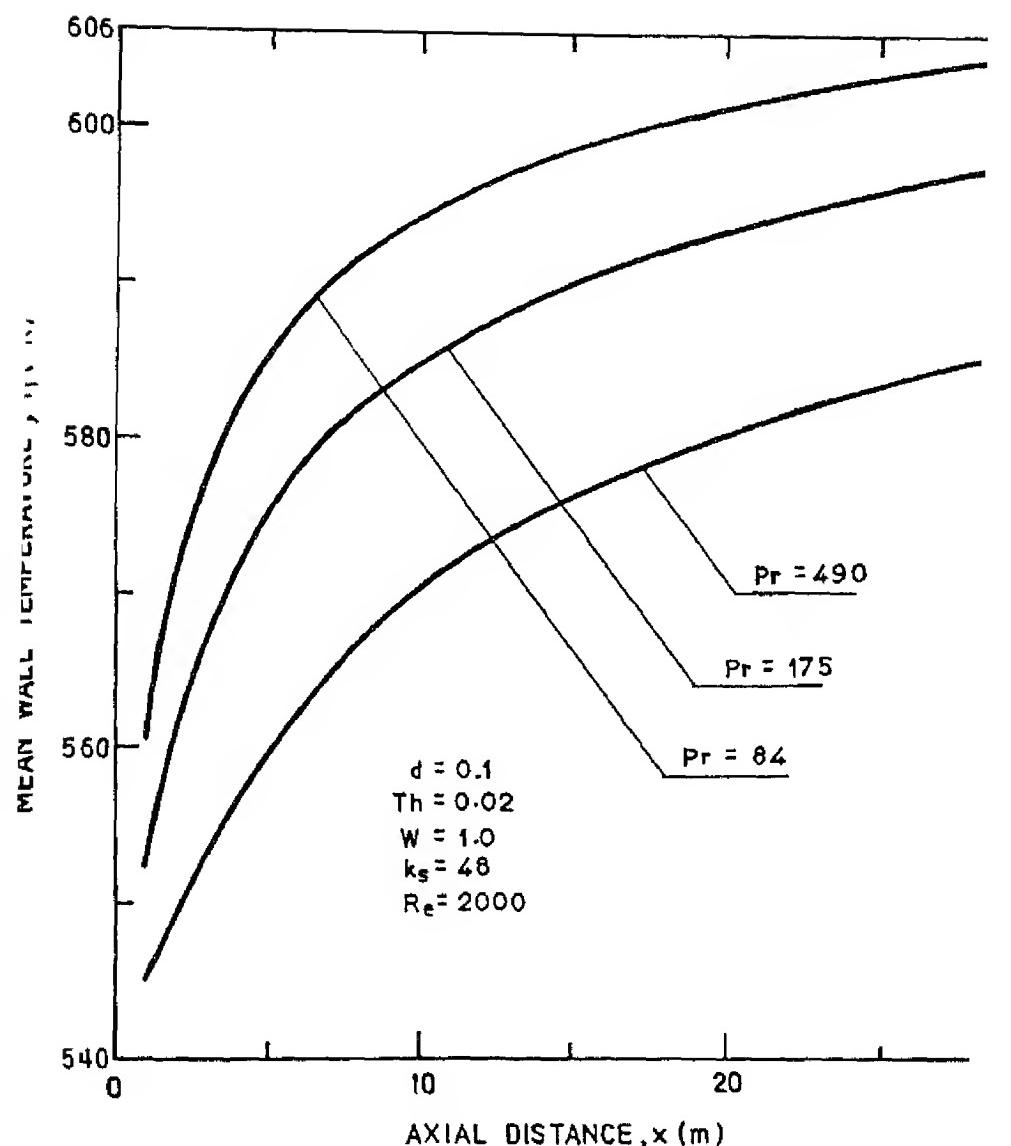


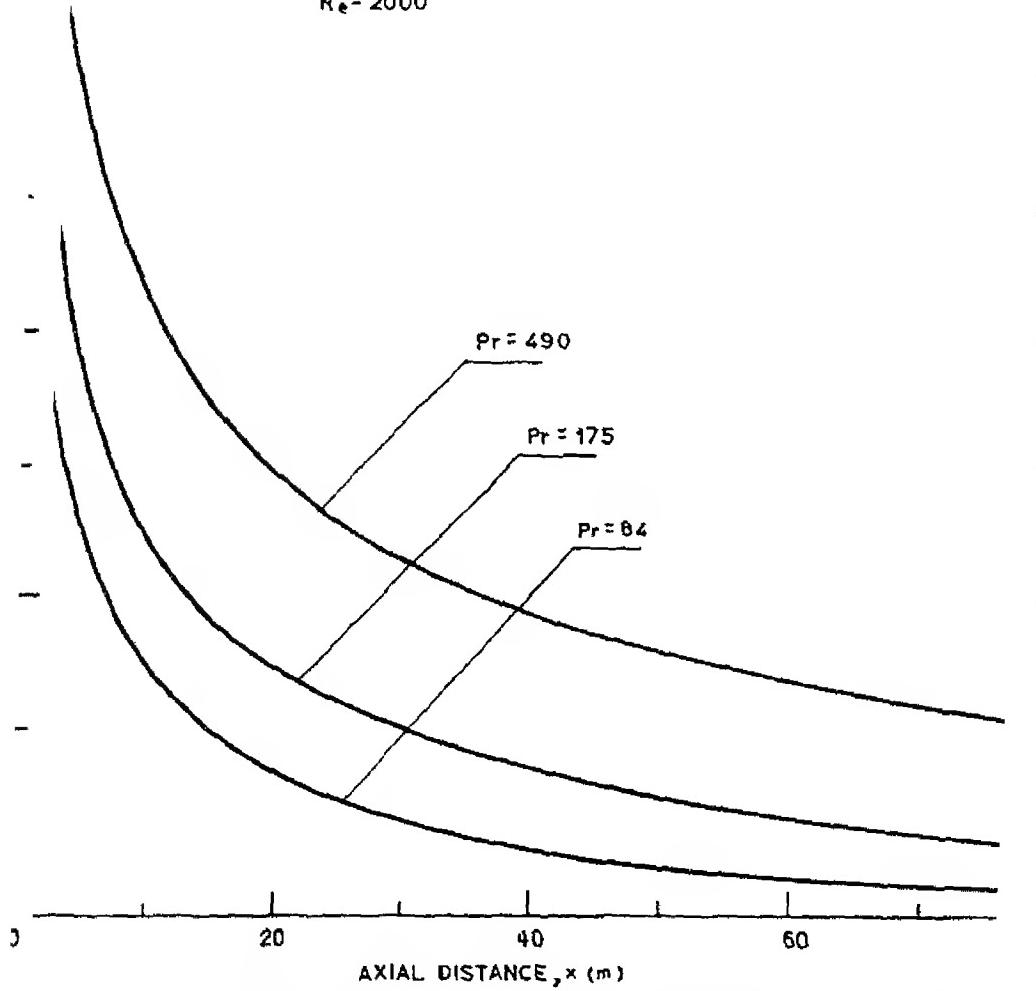
FIG.9 AXIAL DISTRIBUTION OF MEAN WALL TEMPERATURE  
FOR DIFFERENT PRANDTL NUMBER (Pr) AT A CONSTANT  
REYNOLDS NUMBER

Prandtl number fluid and the temperature gradient near the entrance of the tube is more in case of low Prandtl number fluid. This is because for low Prandtl number fluid, the thermal diffusion is much more as compared with high Prandtl number fluids. But, as the axial distance increases i.e. at larger  $x$ , the temperature gradient is not affected much by Prandtl number as the boundary layer thickness grows along the  $x$ -direction.

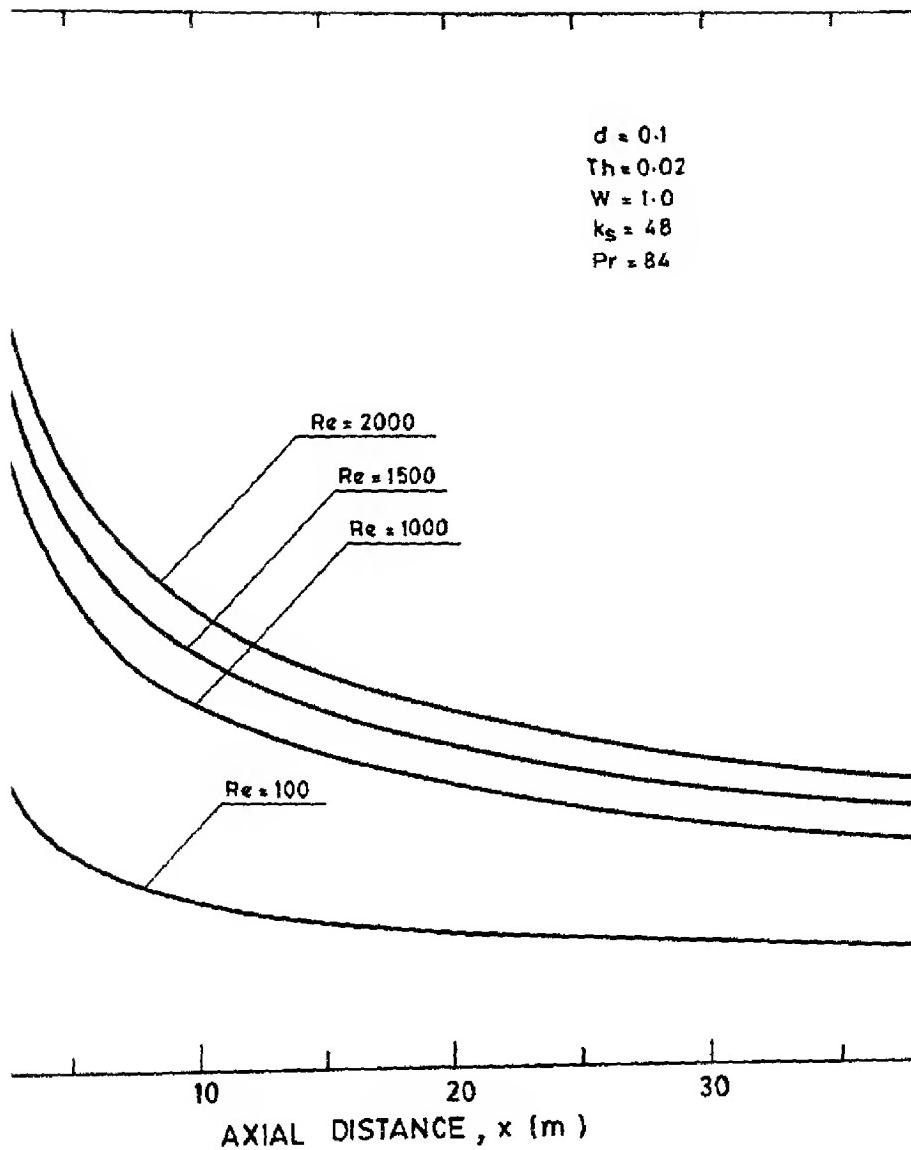
Figure 10 shows the local Nusselt number,  $Nu_x$ , variation along the axial distance,  $x$  for various Prandtl number fluids, Reynolds number remaining same. As expected, Nusselt number is very high at the entrance of the tube and then it decreases monotonically along  $x$  until it becomes almost constant far downstream. Local Nusselt numbers are less for lower Prandtl number fluids. This is because lower Prandtl number fluid have higher thermal diffusivity and hence convective heat transfer is less. So, the heat transfer coefficient is also less which in turn implies that Nusselt number is less. Also, it is seen that thermal entry lengths for higher Prandtl number fluids are much more as they have lower thermal diffusivity which means that thermal boundary layer develops slower.

Figure 11 shows how local Nusselt number,  $Nu_x$  changes along ' $x$ ' for various Reynolds number, Prandtl number remaining same. For higher Reynolds number of the flow, Nusselt numbers are larger. This is because, higher Reynolds number means higher average velocity of the flow and hence more heat is carried away by the fluid and since Nusselt number is the indicator of heat transfer, this results in higher Nusselt nu-

$d = 0.1$   
 $Th = 0.02$   
 $W = 1.0$   
 $k_s = 48$   
 $Re = 2000$



O AXIAL DISTRIBUTION OF LOCAL NUSSELT NUMBER FOR VARIOUS PRANDTL NUMBER ( $Pr$ ) AT CONSTANT  $Re$



AXIAL DISTRIBUTION OF THE LOCAL NUSSELT NUMBER FOR DIFFERENT REYNOLDS NUMBER ( $Re$ ) OF THE FLOW, AT A FIXED VALUE OF  $Th$

Figure 12 shows plots of fluid bulk temperature ( $T_{fb}$ ) along the axial distance ( $x$ ) for various Reynolds numbers, Prandtl number remaining same. The plots indicates that fluid bulk temperature increases with the axial distance,  $x$ . For higher Reynolds number of the flow, the fluid bulk temperatures are less than that for lower Reynolds number. This is because for higher Reynolds number, the average velocity of the flow is higher which results in more convective heat transfer giving rise to lower bulk temperature of the fluid at a given  $x$ -location.

Figure 13 shows plots of fluid bulk temperature ( $T_{fb}$ ) along the axial distance ( $x$ ) for various Prandtl number fluids, Reynolds number of flow remaining same. The plots indicate that fluid bulk temperature increases with axial distance,  $x$ . Also, for higher Prandtl number, fluid bulk temperatures are lower. This is because for higher Prandtl number fluids thermal diffusivity is lower which in turn results in higher convective heat transfer and hence lower fluid bulk temperature at a given  $x$ -location.

Finally, the accuracy of the present numerical solution is checked by comparing the Nusselt number for the full developed condition when constant heat flux condition is imposed on the periphery of the tube and conduction in the tube wall is neglected, i.e. the tube is considered very very thin. Nusselt number at a distance of 2000 meters along the tube is found to be 4.5 which compares very well with 4.364, the Nusselt number for fully developed constant heat flux

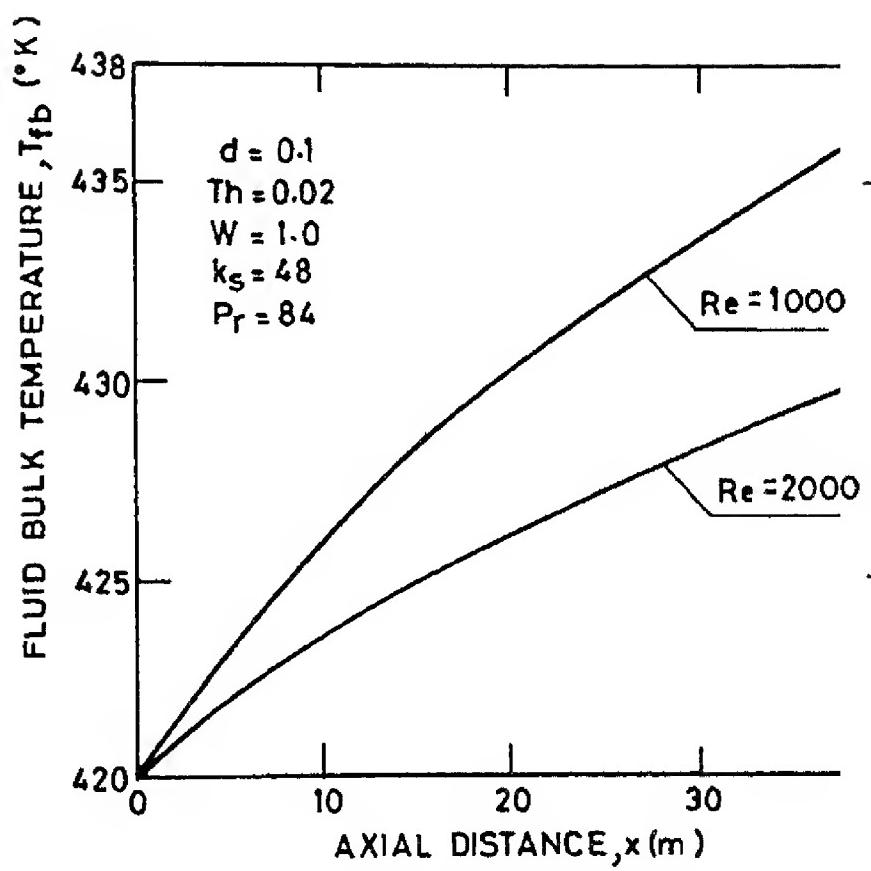


FIG.12 AXIAL DISTRIBUTION OF FLUID BULK TEMPERATURE FOR VARIOUS VALUES OF  $\text{Re}$  OF

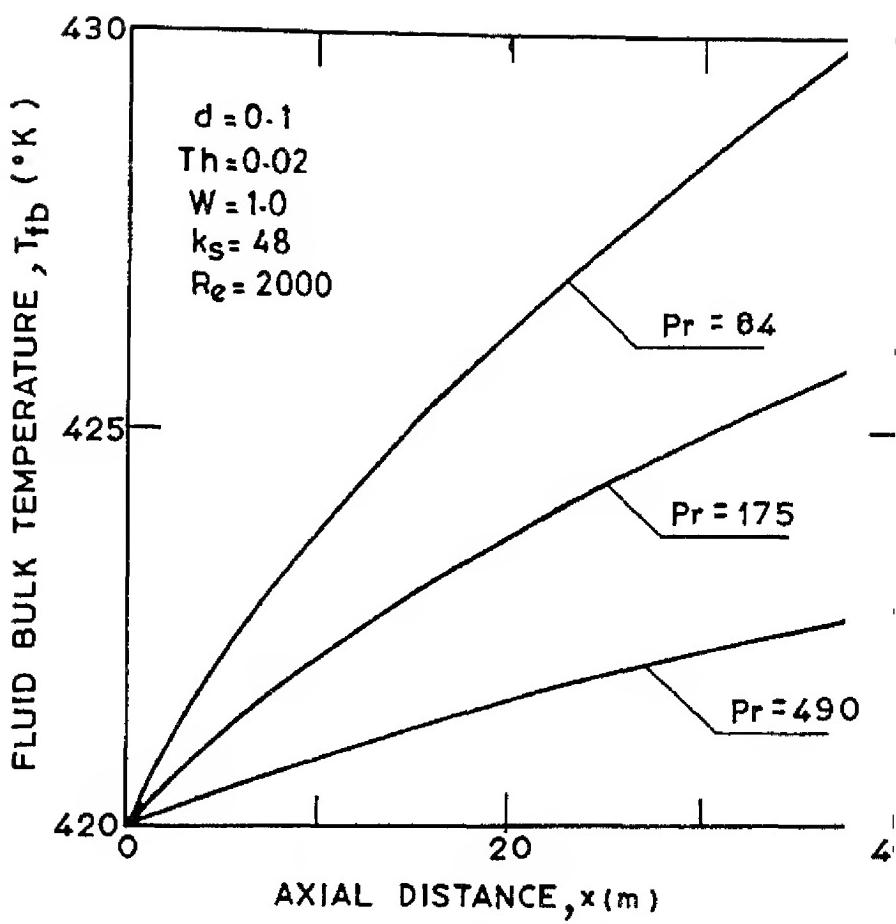


FIG. 13 AXIAL DISTRIBUTION OF THE FLUID BULK TEMPERATURE FOR DIFFERENT VALUES OF PRANDTL (Pr)

condition as obtained by analytical solution. Since, theoretically thermally fully developed condition occurs at infinity, it is not numerically possible to go to that extent because of limits on computation time. As the local Nusselt number for uniform heat flux condition obtained numerically is very much closer to the value obtained by analytical solution, the present solution for non-uniform heat flux condition can be called accurate enough.

## CHAPTER 5

### CONCLUSIONS

A simple but computationally efficient numerical scheme has been used to solve the problem of conjugate heat transfer in a laminar flow of high Prandtl number fluid in a circular tube whose periphery is exposed to non-uniform radiation heat flux from a large heated wall. The numerical results obtained agree very well with the physical expectations. Also, the accuracy of the present numerical solution has been checked by comparing the Nusselt number obtained numerically for thermally fully developed condition with that obtained analytically for the idealised case of constant heat flux condition on the periphery of a very thin tube. Also, this analysis can be modified to take into account variable property fluid and the tube wall. The present analysis is for infinite length of the tube, but it can also be extended to the tube of finite length with necessary change in boundary conditions.

REFERENCES

Reynolds, W.C., "Heat Transfer to Fully Developed Laminar Flow in a Circular Tube with Arbitrary Circumferential Heat Flux", Trans. ASME, Vol. 82, 1960, pp. 108-112.

Reynolds, W.C., "Turbulent Heat Transfer in a Circular Tube with Variable Circumferential Heat Flux", International Journal of Heat and Mass Transfer, Vol. 6, 1963, pp. 445-454.

Faghri, M. and Sparrow, E.M., "Simultaneous Wall and Fluid Axial Conduction in Laminar Pipe-Flow Heat Transfer", ASME Journal of Heat Transfer, Vol. 102, 1980, pp. 58-63.

Zariffch, E.K., Soliman, H.M. and Trupp, A.C., "The Combined Effects of Wall and Fluid Axial Conduction on Laminar Heat Transfer in Circular Tubes", Paper FC-23, Proceedings, Seventh International Heat Transfer Conference, Vol. IV, 1982, pp. 131-136.

Campo, A. and Rangel, R., "Lumped-System Analysis for the Simultaneous Wall and Fluid Axial Conduction in Laminar Pipe-Flow Heat Transfer", Physico Chemical Hydrodynamics, Vol. 4, 1983, pp. 163-173.

Davis, E.J. and Venkatesh, S., "The Solution of Conjugated Multiphase Heat and Mass Transfer Problems", Chemical Engineering Science, Vol. 34, 1979, pp. 775-787.

Luikov, A.V., Aleksashenko, V.A. and Aleksashenko, A.A., "Analytical Methods of Solution on Conjugated Problems in Convective Heat Transfer", International Journal of Heat and Mass Transfer, Vol. 14, 1971, pp. 1047-1056.

Patankar, S.V., "A Numerical Method for Conduction in Composite Materials, Flow in Irregular Geometries and Conjugate Heat Transfer", Paper CO-14, Proceedings, Sixth International Heat Transfer Conference, Vol. III, 1978, pp. 297-302.

Barozzi, G.S. and Pagiliarini, G., "A Method to Solve Conjugate Heat Transfer Problems: The Case of Fully Developed Laminar Flow in a Pipe", ASME Journal of Heat Transfer, Vol. 107, 1985, pp. 77-83.

Patankar, S.V., Numerical Heat Transfer and Fluid Flow, Hemisphere, Washington, D.C., 1980.

Siegel, R. and Howell, J.R., Thermal Radiation Heat Transfer, International Student Edition, McGraw-Hill, 19

## APPENDIX A

### DETERMINATION OF THE CONFIGURATION FACTOR

#### A.1 Definition

The configuration factor is also called the geometric view factor. For any two given surfaces, the orientation between them affects the fraction of the radiation energy leaving one surface that strikes the other surface directly. Therefore, the orientation of the surfaces plays an important role in radiation heat exchange. The physical significance of the view factor between two surfaces is that it represents the fraction of radiative energy leaving one surface that strikes the other surface directly.

#### A.2 Hottel's Cross-String Method [11]

Consider an enclosure, as shown in Figure A-1, consisting of four surfaces which are very long in the direction perpendicular to the plane of the figure. The surfaces can be flat, convex or concave. To find the view factor,  $F_{1-2}$ , imaginary strings are assumed, shown by dashed line in the figure, they are tightly stretched among the four corners A, B, C, D of the enclosure. If  $L_i$  ( $i = 1, 2, 3, 4, 5, 6$ ) denote the lengths of the strings joining the corners, as illustrated in Figure A-1. According to Hottel, the view factor  $F_{1-2}$  can be expressed as,

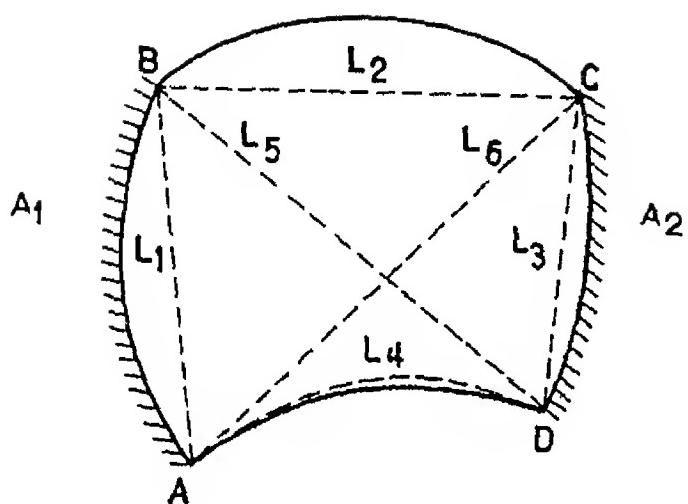


FIG. A-1 DETERMINATION OF CONFIGURATION FACTOR  
BETWEEN THE SURFACES  $A_1$  AND  $A_2$  BY  
HOTTEL'S CROSS-STRING METHOD

98361

$$F_{1-2} = \frac{\text{Total crossed length} - \text{Total uncrossed length}}{2 \times \text{Length of surface '1'}}$$

$$= \frac{(L_5 + L_6) - (L_2 + L_4)}{2L_1}, \quad \begin{matrix} \text{Length of surface '1'} \\ \text{approximated to } L_1 \end{matrix}$$

### A.3 Derivation of the Equations for Determining Configuration Factor

For the present case the configuration factor is found between any point of the outer surface of the tube and the large heated wall placed over the tube. Each point on the upper surface of the tube receives radiation heat flux from the entire plate.

Figure A-2 is considered, the tube is placed at a distance  $d$  vertically below the large wall. Both the wall and the tube are infinitely long in the direction perpendicular to the plane of the paper.

In Figure A-2, A is point of tangent on the outer circle from the right end point N, of the wall. Similarly A' is the touching point of the tangent drawn on the outer circle from the left end point, N', of the wall.

Clearly the region below the points A and A' do not get any radiation heat flux directly from the hot plate. Thereby the configuration factor in this region is obviously zero. So, the location of points A and A' is most important, however it is to be noted that when the tube is placed vertically below any edge then point A or A' will be shifted to point C or C' respectively. Therefore in the Figure A-2 angle  $\alpha$  cannot be less than  $\beta$  and limiting value of  $\alpha$  is  $\beta$ ,

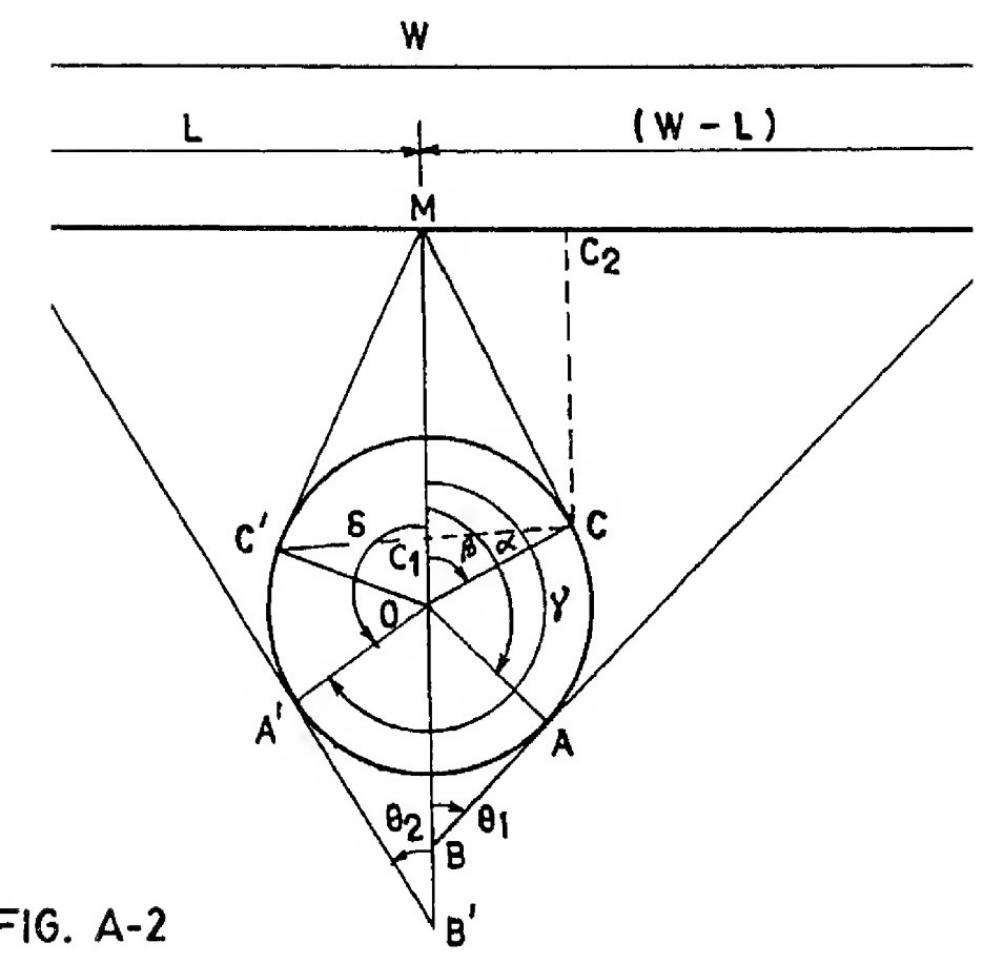


FIG. A-2

however if the top point of the tube, P is placed not directly below the plate then  $\alpha$  can be less than  $\beta$ .

To find out  $\beta$  the triangle MCO is considered,

$$MO = d + r_2$$

$$OC = r_2 \text{ = Outer radius}$$

$$\therefore MC = \sqrt{OM^2 - OC^2}, \text{ as } MCO \text{ is a right angle triangle and } \angle MCO = 90^\circ$$

$$\text{or } MC = \sqrt{(d + r_2)^2 - r_2^2} = \sqrt{d^2 + 2dr_2}$$

$$\text{As, } \angle MOC = \beta, \therefore \tan \beta = \frac{MC}{OC} = \frac{\sqrt{d^2 + 2dr_2}}{d}$$

$$\therefore \beta = \tan^{-1}\left(\frac{\sqrt{d^2 + 2dr_2}}{d}\right) \quad (\text{A.1})$$

Consider the triangle OAB,

$$\angle AOB = (180^\circ - \alpha)$$

$$\angle OAB = 90^\circ$$

$$\therefore \theta_1 = 180^\circ - [\angle AOB + \angle OAB] = \alpha - 90^\circ$$

In the right angle triangle OAB

$$\frac{OA}{OB} = \sin \theta_1 = \sin(\alpha - 90^\circ) = -\cos \alpha$$

$$\therefore OB = -OA/\cos \alpha = -r_2/\cos \alpha$$

$$\therefore MB = MO + OB = d + r_2 - r_2 \sec \alpha$$

$$MN = W - L$$

$$\therefore \frac{MN}{MB} = \tan \theta_1 = \tan(\alpha - 90^\circ) = -\cot \alpha$$

$$\text{or, } \frac{W - L}{d + r_2 - r_2 \sec \alpha} = -\cot \alpha$$

$$\text{or, } (W - L)\sin \alpha = - (d + r_2)\cos \alpha + r_2$$

Squaring both sides of the above expression and simplifying it the following expression is obtained

$$A \cos^2 \alpha - B \cos \alpha + C = 0.$$

$$\text{where } A = (W - L)^2 + (d + r_2)^2$$

$$B = 2r_2(d + r_2)$$

$$C = r_2^2 - (W - L)^2$$

$$\therefore \cos \alpha = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{B \pm D}{2A}, \quad D = \sqrt{B^2 - 4AC}$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \frac{1}{2A} \sqrt{4A^2 - (B + D)^2}$$

$$\text{when, } \cos \alpha = \frac{B + D}{2A}$$

$$= \frac{1}{2A} \sqrt{4A^2 - (B - D)^2}$$

$$\text{when, } \cos \alpha = \frac{B - D}{2A}$$

$$\therefore \tan \alpha = \frac{\sqrt{4A^2 - (B - D)^2}}{B - D} \quad (\text{A.2})$$

$$\text{or, } = \frac{\sqrt{4A^2 - (B + D)^2}}{B + D}$$

Similarly, to find out  $\delta$ , the triangle  $OA'B'$  is considered.

In triangle  $OA'B'$ ,

$$\angle OB'A' = \theta_2, \quad \angle B'OA' = (180^\circ - \delta)$$

$$\therefore \theta_2 = 180^\circ - [90^\circ + (180^\circ - \delta)] = (\delta - 90^\circ)$$

$$OB' = OA'/\sin \theta_1 = -r_2/\cos \delta$$

From triangle,  $MB'N'$ ,

$$MB' = MO + OB' = d + r_2 - r_2 \sec \delta$$

$$MN' = L$$

$$\tan \theta_2 = \frac{MN'}{MB'} = \frac{L}{(d + r_2 - r_2 \sec \delta)}$$

$$\text{or, } -\cot \delta = \frac{L}{d + r_2 - r_2 \sec \delta}$$

Solving this equation for ' $\delta$ ',

$$\begin{aligned} \tan \delta &= \frac{\sqrt{4A_1^2 - (B_1 + D_1)^2}}{B_1 + D_1} \\ \text{or, } &= \frac{\sqrt{4A_1^2 - (B_1 - D_1)^2}}{B_1 - D_1} \end{aligned} \tag{A.3}$$

$$\text{where, } D_1 = \sqrt{(B_1^2 - 4A_1 C_1)}$$

$$A_1 = L^2 + (d + r_2)^2$$

$$B_1 = 2r_2(d + r_2)$$

$$C_1 = r_2^2 - L^2$$

$$\text{Also, } \gamma = 2\pi - \delta$$

(A.4)

From the equations (A.1), (A.2), (A.3), (A.4) the values of  $\beta$ ,  $\alpha$ ,  $\delta$  and  $\gamma$  are obtained, which play important role for the calculation of configuration factor.

Next step is to express the configuration factor,  $F_{2W}$  as a function of ' $\phi$ '. Refer to Figure A-3.

Let, the configuration factor for the elemental arc XY on the upper surface is required to be found. The length XY is sufficiently small such that in the limiting case this element turns to a point and the expression for  $F_{2W}$  for this element becomes the value of  $F_{2W}$  at the point P.

Points X and Y are joined with points M and N by four strings, two crossed, two uncrossed, as illustrated in the Figure A-3. Here,  $XN$  and  $YN'$  are the uncrossed strings and  $XN'$  and  $YN$  are the crossed strings.

By Hottel's Cross-String method, as described earlier,

$$\begin{aligned} F_{2W} \text{ at } P &= \lim_{XY \rightarrow 0} \frac{(XN' + YN) - (XN + YN')}{2XY} \\ &= \lim_{XY \rightarrow 0} \frac{(XN' - YN') + (YN - XN)}{2XY} \end{aligned}$$

From the Figure A-3,  $XN' - YN' = XY$

and  $YN - XN \approx YZ$

where  $XZ$  is the perpendicular drawn on  $YN$  from X

$$F_{2W} \text{ at } P = \lim_{XY \rightarrow 0} \frac{XY + YZ}{2XY}$$

This equation is valid for the right half of the tube, moreover upto  $\phi = \alpha$ .

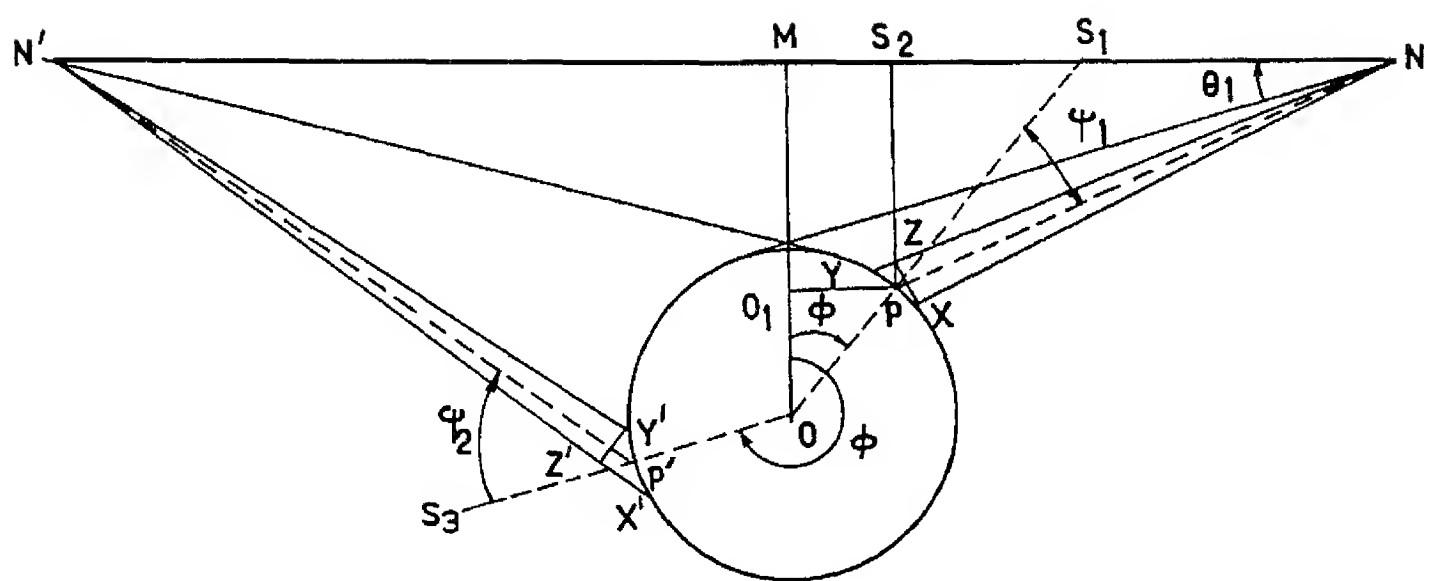


FIG. A-3

For  $\theta > \alpha$ ,  $F_{2W} = 0$ , till  $\theta = \gamma$  in the other half.

To find out YZ, the enlarged view of the triangle XYZ is considered, as shown in Figure A-4.

From this figure,  $\angle NPS_1 = \Psi_1$ ,  $\angle YPO = 90^\circ$  as  $OS_1$  is normal to the surface at point P.

XY is very small, so that YN and PN are almost parallel, and as XZ is perpendicular to YN, so XZ is also perpendicular to PN, i.e.  $\angle PX_1X = 90^\circ$  in the triangle  $PX_1X$  and as  $OS_1$  is perpendicular to XY,  $\angle XPS_1 = 90^\circ$ .

$$\therefore \angle NPX = 90^\circ - \Psi_1$$

$$\therefore \angle X_1XP = 180^\circ - [\angle NPX + \angle PX_1X] = \Psi_1$$

Now from the right angle triangle YZX,

$$\frac{YZ}{XY} = \sin \Psi_1, \text{ i.e. } YZ = XY \sin \Psi_1$$

$$\begin{aligned} \text{So, } F_{2W} \text{ at P or } F_{2W}(\theta) &= \lim_{XY \rightarrow 0} \frac{XY + YZ}{2XY} = \lim_{XY \rightarrow 0} \frac{XY + XY \sin \Psi_1}{2XY} \\ &= \frac{1}{2} (1 + \sin \Psi_1) \end{aligned}$$

$$\therefore F_{2W}(\theta) = \frac{1}{2} (1 + \sin \Psi_1), \text{ for } 0 \leq \theta \leq \alpha \quad (\text{A.5})$$

$\Psi_1$  is found out considering Figure A-4,

$$\begin{aligned} \text{From triangle } PS_1N, \quad \Psi_1 &= \pi - (\theta_1 + \pi/2 + \theta) \\ &= \frac{\pi}{2} - (\theta_1 + \theta) \end{aligned} \quad (\text{A.6})$$

$$\text{From triangle } PS_2N, \quad \tan \theta_1 = \frac{PS_2}{S_2N}$$

$$OM = d + r_2, \quad MS_2 = r_2 \sin \theta, \quad MN = (W - L),$$

$$OO_1 = r_2 \cos \theta, \quad S_2N = MN - MS_2 = (W - L) - r_2 \sin \theta$$

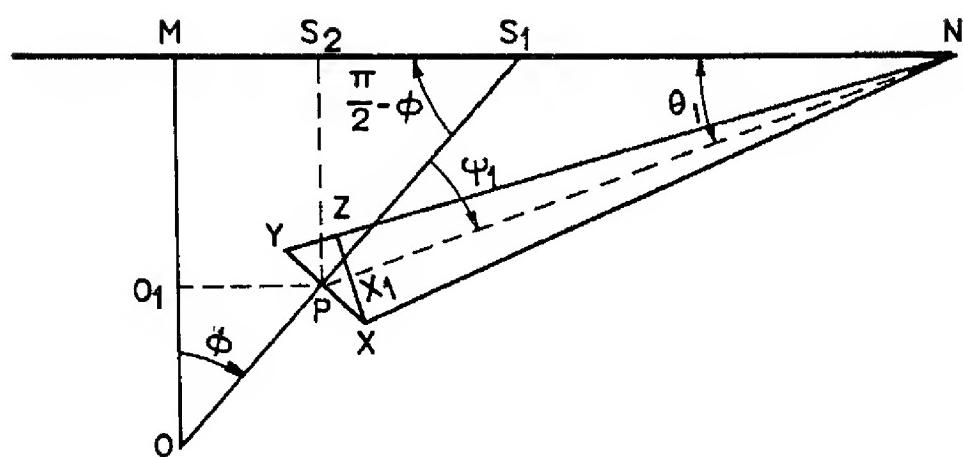


FIG. A-4

$$\begin{aligned}
 PS_2 &= OM - OO_1 = (d + r_2) - r_2 \cos\theta \\
 \therefore \tan \theta_1 &= \frac{(d + r_2) - r_2 \cos\theta}{(W - L) - r_2 \sin\theta} \\
 \therefore \theta_1 &= \tan^{-1} \left[ \frac{(d + r_2) - r_2 \cos\theta}{(W - L) - r_2 \sin\theta} \right] \quad (A.7)
 \end{aligned}$$

For the left half, i.e. for  $\gamma \leq \theta \leq 2\pi$ , Figures A-3 and A-5 are considered.

The uncrossed strings are  $X'N'$  and  $Y'N$ , and the crossed strings are  $X'N$  and  $Y'N'$ , all are joined clockwise as in the case of right half of the tube.

As  $\theta$  is measured in the clockwise direction, the relative positions of points  $X'$  and  $Y'$  must be noted carefully.

$$F_{2W \text{ at } P'} = \lim_{X'Y' \rightarrow 0} \frac{(X'N + Y'N') - (X'N' + Y'N)}{2X'Y'}$$

Using  $X'N - Y'N = X'Y'$ , and  $X'N' - Y'N' \approx X'Z'$

$$F_{2W \text{ at } P'} = \lim_{X'Y' \rightarrow 0} \frac{X'Y' - X'Z'}{2X'Y'}$$

Figure A-5 is considered to find the expression of  $F_{2W}$  at  $P'$

$$\text{As } \angle Y'P'S_3 = 90^\circ, \quad \angle Y'P'Z'_1 = 90^\circ - \Psi_2$$

Again  $Y'Z'$  is the normal drawn on  $X'N'$  and for a very small element  $X'Y'$ ,  $P'N'$  and  $X'N'$  are almost parallel. This gives

$$\angle P'Z'_1Y' = 90^\circ$$

$$\angle Z'Y'P' = \pi - (\angle Y'P'Z'_1 + \angle P'Z'_1Y') = \Psi_2$$

In the right angle triangle  $Y'X'Z'$ ,

$$\angle X' = 90^\circ - \Psi_2$$

$$X'Z' - X'Y' \cos(90^\circ - \Psi_2) = X'Y' \sin \Psi_2$$

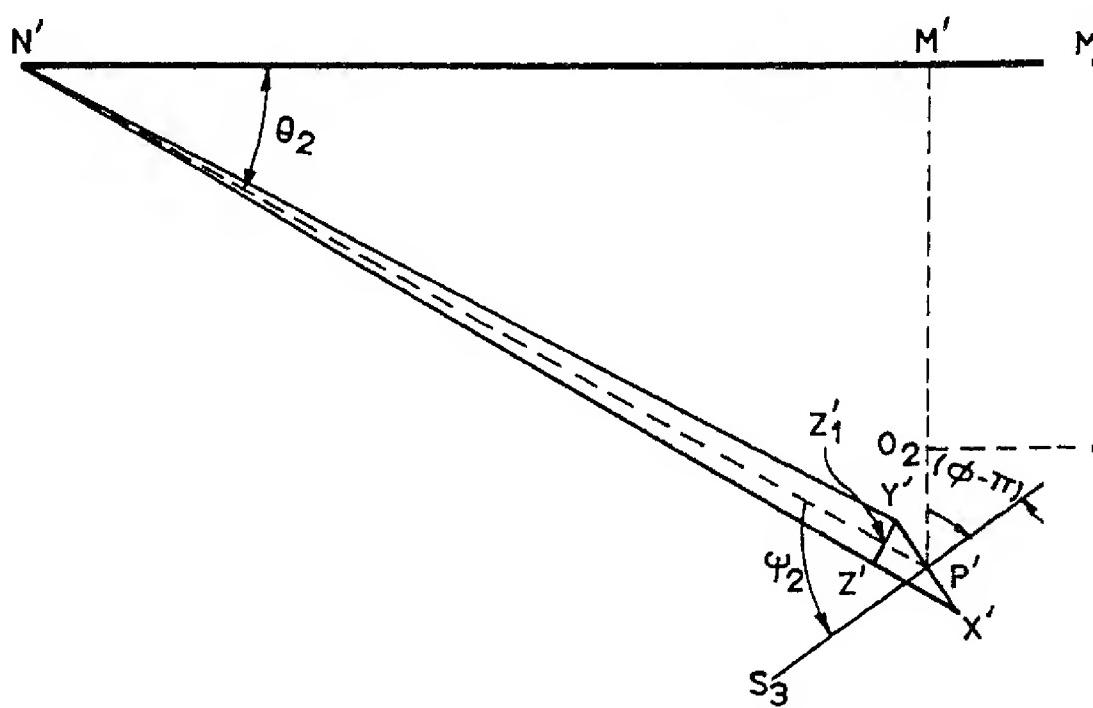


FIG. A-5

From expression of  $F_{2W}$  at  $P'$ ,

$$F_{2W} \text{ at } P' = \lim_{X'Y' \rightarrow 0} \frac{X'Y' - X'Y' \sin \Psi_2}{X'Y'}$$

$$\text{or, } F_{2W} \text{ at } P' = F_{2W}(\emptyset) = \frac{1}{2} (1 - \sin \Psi_2) \quad (\text{A.8})$$

From the same figure,  $\angle OP' O_2 = (\emptyset - \pi)$

$$\begin{aligned} M'P'N' &= \angle OP'N' - \angle OP'O_2 \\ &= (\pi - \Psi_2) - (\emptyset - \pi) = 2\pi - (\emptyset + \Psi_2) \end{aligned}$$

$$\text{From triangle } N'M'P', \quad M'P'N' = \frac{\pi}{2} - \Theta_2$$

Equating these two,

$$\begin{aligned} 2\pi - (\emptyset + \Psi_2) &= \frac{\pi}{2} - \Theta_2 \\ \text{or, } \Psi_2 &= 1.5\pi + \Theta_2 - \emptyset \end{aligned} \quad (\text{A.9})$$

$$\text{where, } \Theta_2 = \tan^{-1} \frac{P'M'}{M'N'}$$

$$\begin{aligned} P'M' &= M'O_2 + O_2P' = d + r_2 + r_2 \cos(\emptyset - \pi) \\ &= d + r_2 - r_2 \cos\emptyset \end{aligned}$$

$$M'N' = MN' - MM' = L - r_2 \sin(\emptyset - \pi) = L + r_2 \sin\emptyset$$

$$\Theta_2 = \tan^{-1} \left( \frac{d + r_2 - r_2 \cos\emptyset}{L + r_2 \sin\emptyset} \right) \quad (\text{A.10})$$

Therefore all the expressions for  $F_{2W}(\emptyset)$  for different values of  $\emptyset$  are listed below.

$$\begin{aligned}
 F_{2W}(\phi) &= \frac{1}{2}(1 + \sin \Psi_1), \quad \text{for } 0 \leq \phi \leq \alpha \\
 &= 0 \quad , \quad \text{for } \alpha < \phi < \gamma \\
 &= \frac{1}{2}(1 - \sin \Psi_2), \quad \text{for } \gamma \leq \phi \leq 2\pi
 \end{aligned}$$

where

$$\Psi_1 = \frac{\pi}{2} - (\Theta_1 + \phi),$$

$$\text{and } \Theta_1 = \tan^{-1} \left[ \frac{(d + r_2) - r_2 \cos \phi}{(W - L) - r_2 \sin \phi} \right]$$

$$\Psi_2 = 1.5\pi + \Theta_2 - \phi,$$

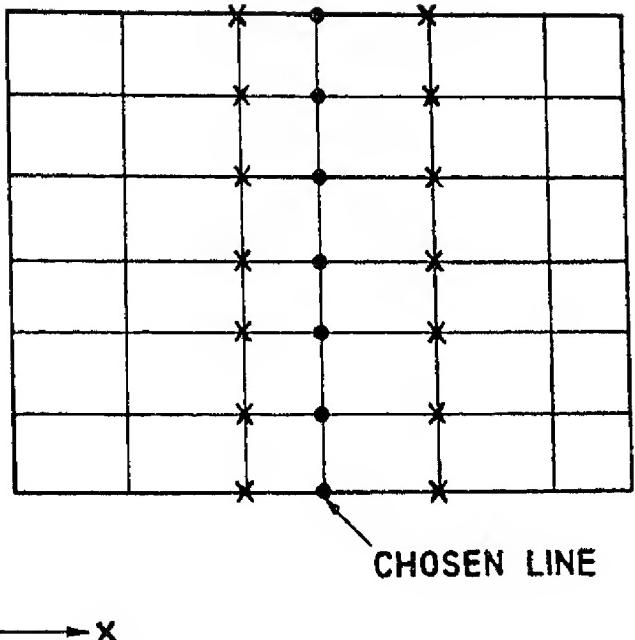
$$\text{and } \Theta_2 = \tan^{-1} \left[ \frac{(d + r_2) - r_2 \cos \phi}{L + r_2 \sin \phi} \right]$$

## APPENDIX B

### THE LINE-BY-LINE METHOD

A line-by-line method is a conventional combination of TDMA (Tri-Diagonal Matrix Algorithm) method for one dimensional situations and the Gauss-Seidel iterative method. According to this method a grid line (in a particular direction) is chosen and the values of temperatures, T's (or any dependent variable) along the neighbouring lines are assumed or known by their latest values, and the T's along the chosen line are solved by TDMA. This procedure is followed for all the lines in the same direction and then the technique is repeated till the required convergence is attained. Sometimes to make the convergence faster the same procedure is repeated for the lines in other direction.

To visualize the line-by-line method more clearly the Figure B-1 is considered. The discretization equations for the grid points along a particular line (either in x-direction or in y-direction) are considered. These equations contain the temperatures at the grid points (shown by crosses) along the two neighbouring lines. If these temperatures are substituted from their latest values, the equations for the grid points (shown by dots) along the chosen line would look like one dimensional equations and could be solved by TDMA. This procedure is carried out for all the lines in y-direction and may be followed by a similar treatment for x-direction.



## 1 REPRESENTATION OF THE LINE-BY-LINE METHOD

The convergence of the line-by-line method is faster because the boundary condition information from the ends of the line is transmitted at once to the interior of the calculation domain, no matter how many grid points lie along the line. The sweep direction (i.e. the sequence in which lines are chosen) plays an important role in the convergence of the iterative technique. A sweep from upstream to downstream would produce much faster convergence than a sweep against the stream.

## APPENDIX C

INPUT DATA TO THE PROGRAM

i) Geometry of the tube:

$r_1$ (m)	$Th = (r_2 - r_1)$ , (m)
	0.005
0.12	0.01
	0.02
	0.03

ii) Position of the tube:

$d$ (m)	$w$ (m)	$L$ (m)
0.07	0.4	
0.10	1.0	$\frac{w}{2}$
0.13	20.0	

iii) Conductivity of the tube material,  $k_s$  ( $\frac{W}{m \cdot K}$ ):

$$k_s = 35, 48 \text{ and } 55.$$

iv) Known temperatures ( $^{\circ}K$ )

$$T_e = 420.0, T_s = 300.0, T_w = 830.0$$

v) Radiation parameters:

$$\sigma = 0.567 \times 10^{-7} \text{ W/(m}^2 \cdot \text{K}^4\text{)}, \epsilon = 0.79$$

vi) Flow parameter:

$$Re = 100, 1000, 1500 \text{ and } 2000$$

where,

$$Re \text{ is defined as: } Re = \frac{\rho (2r_1)}{\nu}$$

vii) Number of grid points taken for numerical solution:

$$N_x = 50, N_{rf} = 30, N_\phi = 40$$

## viii) Properties of fluids:

Pr	$k_f$ $\frac{W}{m \cdot K}$	$\alpha$ $\frac{m^2}{s} \times 10^7$	$\nu$ $\frac{m^2}{s} \times 10^6$
51	0.260	0.932	4.75
84	0.132	0.663	5.60
175	0.135	0.710	12.40
490	0.138	0.769	37.50

## APPENDIX D

## LISTING OF THE COMPUTER PROGRAM

## NOMENCLATURE :

A	= coefficient of TDM	NR	= $N_r$
AK	= $k_s$	NRF	= $N_{rf}$
AKF	= $k_f$	NPR	= $P_r$
AL	= L	PI	= $\pi$
ALPHA	= $\alpha$	QB	= $\bar{q}_1$
ANU	= Nu	R1	= $r_1$
ANUF	= $\nu$	R2	= $r_2$
B	= coefficient of TDM	RENOLD	= Re
C	= coefficient of TDM	RI	= $r_i$
D	= coefficient of TDM	SIG	= $\sigma$
D4	= d	T	= T
DPHI	= $\Delta\Phi$	TFB	= $T_{fb}$
DR	= $\Delta r_s$	TFP	= $T_p$
DRF	= $\Delta r_j$	TS	= $T_s$
DX	= $\Delta x$	TW	= $T_w$
EPS	= $\epsilon$	TWB	= $\bar{T}_1$
EPSN	= Accuracy factor for convergence check	U	= u
F2W	= $F_{2w}$	UAVER	= $\bar{u}$
H	= h	V	= final soln. vector of TDMA
I	= i	W	= w
J	= j		
K	= k		

\*\*\*\*\*

## M. TECH THESIS

TITLE: CONJUGATE HEAT TRANSFER IN A  
 LAMINAR PIPE FLOW SUBJECTED TO  
 NON-UNIFORM RADIATION FROM A LARGE  
 HEATED WALL : THE CASE OF THERMALLY  
 UNDEVELOPED FLOW

\*\*\*\*\*

## MAIN PROGRAM

\*\*\*\*\*

COMMON/AREA1/D4,R2,AL,W,F2W

COMMON/AREA2/A,B,C,D,V

DIMENSION T(50,50),A(50),B(50),C(50),D(50),V(50),F2W(50)

DIMENSION TIP(50,50),T1(50),S(50),U(50),RI(50),TWALL(50)

DIMENSION Q(50)

OPEN(UNIT=21,DEVICE='DSK',FILE='MASTER.OUT')

AL=0.5;W=1.0;R1=0.12;R2=0.14;D4=0.1;

AK=48.0;TS=300.0;SIG=0.567E-07;TW=830.0;EPS=0.79

AKF=0.132;NRF=30

NFM1=NRF-1

ALPHA=0.663E-07

RENOULD=1500.0;NPR=84;ANUF=0.056E-04

UAVER=RENOULD\*0.5\*ANUF/R1

TYPE \*,UAVER

WRITE(21,3333)NPR,RENOULD,ANUF,UAVER

```

3333 FORMAT(2X,I6,2X,F6.1,2X,E8.3,2X,F10.4//)
      DRF=R1/FLOAT(NRF-1)

      PI=3.1415927;NPHT=40;NR=50

      TPI=2.*PI
      DPHI=TPI/FLOAT(NPHT)

      DR=(R2-R1)/FLOAT(NR-NRF)
      P1=2.*R1*DRI+DRF*DRF
      P2=2.*R1*DR-DR*DR
      R=P1*AR/(P2*ARF)
      RR=(R*DR*DR+DRF*DRF)/(DPHI*DPHT)
      A1=2.*R1*R1
      A3=A1*R
      A2=-(A3+A1+2.*RR)
      S2=2.*DR*SIG*EPS/AR
      S2=S2*(2.*R2*R2+R2*DR)
      DX=1.0
      DRDP=DR/DPHI
      DRDPF=DRF/DPHT
      EPSN=0.1

      DO 10 I=1,NRF
      RIC(I)=DRF*(I-1)
10      U(I)=1.0-RIC(I)*R1(I)/(R1*R1)
      DO 15 I=NRF+1,NR
      RT(I)=Rf+DR*(I-NRF)
15      DO 205 J=1,NR
      DO 205 J=1,NPHT
205     TFP(I,J)=420.0
      CALL CONFIG(1,NPHT)

```

```

CONST=4.0*DRF*DRF*UAVR/(ALPHA*DX)
XX=0.0
DO 202 K=1,50
DO 40 L=1,NR
DO 40 J=1,NPHI
40      T(I,J)=450.
DO99 ITER =1,200
TO1 D=T(NR,1)
C
***** J=1 *****
DO 1651 I=2,NRF-1
A(I)=2.*RI(I)*RI(I)-RI(I)*DRF
C(I)=2.*RI(I)*RI(I)+RI(I)*DRF
B(I)=-4.*RI(I)*RI(I)-4.*DRDP*DRDP-CONST*RI(I)*RI(I)*U(I)
1651 D(I)=-CONST*RI(I)*RI(I)*U(I)*TEP(1,1)-2.*DRDP*DRDP*(T(I,2)-
1T(I,NPHI))
A(I)=0.0
B(I)=1.0;C(I)=1.0;D(I)=0.0
DO101 I=NRF+1,NR
A(I)=2.*RI(I)*RI(I)-RI(I)*DR
C(I)=2.*RI(I)*RI(I)+DR*RI(I)
B(I)=-4.*RI(I)*RI(I)-4.*DRDP*DRDP
101 D(I)=-2.*DRDF*URDP*T(I,NPHI)-2.*DRDP*DRDP*T(I,2)
A(NRF)=4.*R2*R2
C(NRF)=0.0
B(NRF)=B(NRF)-4.*S2*T(NR,1)**3.
D(NRF)=D(NRF)-S2*F2W(1)*TW**4.-S2*(1.-F2W(1))*TS**4.-3.*S2
1*T(NR,1)**4.
A(NRF)=A1*B(NRF); A2=C(NRF)-A3

```

```

B(NRF)=-RR*T(NRF,2)-RR*T(NRF,NPH)
CALL TRIDIAG(1,NR)
DO 102 I=1,NR
102 T(I,1)=U(I)
C
      **** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      DO 103 J=2,NPHI-1 **** * * * * * * * * * * * * * * * * * * * * * * * *
      DO 210 T=2,NRF-1
      A(T)=2.*RI(I)*RI(J)-RJ(J)*DRF
      C(I)=2.*RI(I)*RI(I)+RI(I)*DRF
      B(I)=-4.*RI(I)*RI(I)-4.*DRDPF*DRDPF-CONST*RI(I)*RI(I)*U(I)
      D(I)=-CONST*RI(I)*RT(I)*U(I)*TFP(I,J)-2.*DRDPF*DRDPF*(T(I,J+1)-
      IT(I,J-1))
      A(I)=0.0;C(I)=1.0
      B(I)=-1.0;D(I)=0.0
      DO 104 T=NRF+1,NR
      A(T)=2.*RI(I)*RI(I)-RJ(I)*DR
      C(I)=2.*RJ(I)*RI(I)+RJ(I)*DR
      B(I)=-4.*RI(I)*RT(I)-4.*DRDP*DRDP
      IT(I)=-2.*DRDP*DRDP*T(I,J-1)-2.*DRDP*DRDP*T(I,J+1)
      A(NR)=4.*R2*K2
      C(NR)=0.0
      B(NR)=B(NR)-4.*S2*T(NR,J)**3,
      D(NR)=B(NR)-S2*F2W(J)*TW**4,-S2*(1.-F2W(J))*TS**4,-3.*S2
      1*T(NR,J)**4,
      A(NRF)=A1;B(NRF)=A2;C(NRF)=A3
      D(NRF)=-RR*(T(NRF,J+1)+T(NRF,J-1))
      CALL TRIDIAG(1,NR)
      IO 105 I 1,NR

```

```

105      T(J,J)=V(I)
103      CONTINUE
C
* **** * **** * **** * **** * **** * **** * **** * **** * **** *
DO 215 I=2,NRF-1
A(I)=2.*RI(I)*RI(I)-RT(I)*DRF
C(I)=2.*RI(I)*RT(I)+RI(I)*DRF
B(I)=-4.*RI(I)*RT(I)-4.*DRDP*DRDP-CONST*RI(I)*RI(I)*U(I)
215      D(I)=-CONST*RI(I)*RT(I)*U(I)*TEP(T,NPHI)-2.*DRDP*DRDP*(T(I,1)
1+I(I,NPHI-1))
A(1)=0.0;B(1)=-1.0;C(1)=1.0;D(1)=0.0
DO 106 J=NRF+1,NR
A(I)=2.*RI(I)*RI(I)-RT(I)*DR
C(I)=2.*RI(I)*RT(I)+RI(I)*DR
B(J)=-4.*RI(I)*RI(I)-4.*DRDP*DRDP
106      D(J)=-2.*DRDP*DRDP*T(I,NPHI-1)-2.*DRDP*DRDP*T(I,1)
A(NR)=4.*R2*R2
C(NR)=0.0
B(NR)=B(NR)-4.*S2*T(NR,NPHI)**3,
D(NR)=D(NR)-S2*F2W(NPHI)*TW**4,-S2*(1.-F2W(NPHI))*TS**4,-S2*3.*T(NR,NPHI)**4.
A(NRF)=A1;B(NRF)=A2;C(NRF)=A3
D(NRF)=-RR*T(NRF,1)-RR*T(NRF,NPHI-1)
CALL TRIDAG(1,NR)
DO 107 I=1,NR
107      T(I,NPHI)=V(I)
IF(ABS(TOLD-T(NR,1)).LE.EPSN)GO TO 110
99      CONTINUE
WRITE(21,115)

```

```

115      FORMAT(2X,'NOT CONVERGED'//)
          WRITE(21,777)
777      FORMAT(2X,'INTER FACE TEMP. > J=1,NPHI'//)
          WRITE(21,779)(T(NRF,J),J=1,NPHI)
779      FORMAT(8(F12.3))
          WRITE(21,885)
885      FORMAT(2X,'TEMP. FOR THE WHOLE REGION WITH J=1,NPHI,S&I=1,5'//)
          WRITE(21,*)((T(T,I), I=1,NPHI,5), T=1,NR)
          DO 889 I=1,NRI
          DO 889 J=1,NPHI
889      TFP(I,J)=T(T,I)
C      HEAT TRANSFER CALCULATIONS
          DO 11 I=1,NRI
          DO 12 J=1,NPHI
12      TT1(J)=T(T,J)
          TT1(NPHI+1)=TT1(1)
          CALL SIMPSN(0,TPI,NPHI,TT1,XINTGL)
          S(I)=XINTGL*R1(D)*U(I)
11      CONTINUE
          CALL SIMPSN(0,R1,NEM1,0,XINTGL)
          TFB=2.*XINTGL/(PI*R1*R1)
          DO 13 J=1,NPHI
          TWALL(J)=T(NRF,J)
13      Q(J)=(T(NRF+1,J)-T(NRF,J))/DR
          TWALL(NPHI+1)=TWALL(1)
          Q(NPHI+1)=Q(1)
          CALL SIMPSN(0,TP1,NPHI,TWALL,XINTGL)
          TWB=XINTGL/TP1

```

```

CALL SIMPSN(0,TPI,NPHI,Q,XINTGL)
QB=XINTGL/TPI

H=AR*QB/(TWB TFB)
ANU=2.*H*R1/ARF

TYPE *,ANU

XX=XX+DX

WRITE(21,14)XX,TWB,QB,TFB,H,ANU
14 FORMAT(2X,F8.2,4X,F8.3,4X,F7.2,4X,F8.3,4X,F7.3,4X,F8.3)
202 CONTINUE
STOP
END

C *****
C          S U B R O U T I N E : T R I D A G
C
C (THIS SUBROUTINE SOLVES SIMULTANEOUS LINEAR NON-HOMOGENIOUS EQNS.
C BY TRI-DIAGONAL MATRIX SOLUTION)
C *****
C SUBROUTINE TRIDAG(IF,L)
COMMON/AREA2/A,B,C,D,V
DIMENSION BETA(51),GAMMA(51)
DIMENSION A(50),B(50),C(50),D(50),V(50)
C COMPUTE INTERMEDIATE ARRAYS BETA AND GAMMA
BETA(IF)=B(IF)
GAMMA(IF)=D(IF)/BETA(IF)
IFF1=IF+1
DO 1 I=IFF1,L,
BETA(I)=B(I)-A(I)*C(I-1)/BETA(I-1)
1 GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETA(I)

```

```

C      COMPUTE FINAL SOLUTION VECTOR V
V(L)=GAMMA(L)
LAST=L-TF
DO 2 K=1,LAST
I=L-K
V(I)=GAMMA(I)-C(I)*V(I+1)/BETA(I)
RETURN
END

```

\*\*\*\*\*

#### C S U B R O U T I N E : C O N F I G

C SUBROUTINE TO CALCULATE CONFIGURATION FACTOR

\*\*\*\*\*  
SUBROUTINE CONFIG(IS,NPHI)

COMMON/AREA1/D4,R2,AL,W,F2W

DIMENSION F2W(1,0)

PI=3.1415927

PI=2.\*PI

D1=SQRT(D4\*D4+2.\*R2\*D4)

BETA=ATAN(D1/R2)

B1=2.\*R2\*(D4+R2)

C1=R2\*R2-(W-AL)\*(W-AL)

A1=(D4+R2)\*(D4+R2)+(W-AL)\*(W-AL)

D1=SQRT(B1\*B1-4.\*A1\*C1)

D2=4.\*A1\*A1-(D1+B1)\*(D1+B1)

D2=SQRT(D2)

ALPHA1=ATAN(D2/(B1+D1))

```

D2=4.*A1*A1-(D1-B1)*(D1-B1)

D2=SQRT(D2)

ALPHA2=ATAN(D2/(B1-D1))

IF(ALPHA2.LT.0.0)ALPHA2=ALPHA2+PI

IF(ALPHA2.LT.BETA) GO TO5

ALPHA=ALPHA2

GO TO 6

5 ALPHA=ALPHA1

6 C1=R2*R2*AL*AL

A1=AL*AL+(D4+R2)*(D4+R2)

D1=SQRT(B1*B1-4.*A1*C1)

D2=4.*A1*A1-(D1+B1)*(D1+B1)

D2=SQRT(D2)

ALPHA1=ATAN(D2/(D1+B1))

D2=4.*A1*A1-(D1-B1)*(D1-B1)

D2=SQRT(D2)

ALPHA2=ATAN(D2/(B1-D1))

IF(ALPHA2.LT.0.0)ALPHA2=ALPHA2+PI

IF(ALPHA2.LT.BETA) GO TO 7

DELTA=ALPHA2

GO TO 16

7 DELTA=ALPHA1

16 DFHI=TPI/FLOAT(NPHI)

GAMA=TPI-DELTA

PHI=0.0

K=1

10 X=(D4+R2*R2*COS(PHI))/(W-AL-R2*SIN(PHI))

THETA=ATAN(X)

```

```

SY1=0.5*PI-THETA-PHI
F1W=0.5*(1.+SIN(SY1))
F2W(K)=F1W
WRITE(21,*)PHI,F1W
PHI=PHI+DPHI
K=K+1
IF(PHI.LE.ALPHA)GO TO 10
15 F1W=0.0
WRITE(21,*)PHI,F1W
F2W(K)=F1W
K=K+1
PHI=PHI+DPHI
IF(PHI.LE.GAMA)GO TO 15
20 X=(D4+R2-R2*COS(PHI))/(AL+R2*SIN(PHI))
THETA=ATAN(X)
SY1=1.5*PI+THETA-PHI
F1W=0.5*(1.-SIN(SY1))
WRITE(21,*)PHI,F1W
F2W(K)=F1W
K=K+1
PHI=PHI+DPHI
IF(PHI.LE.TPI)GO TO 20
RETURN
END
C ****

```

C THIS SUBPROGRAM INTEGRATES ANY GIVEN FUNCTION OVER A FINITE  
C LIMIT USING SIMPSON'S 1/3- RD RULE  
C \*\*\*\*  
C SUBROUTINE SIMPSON(XMIN,XMAX,N,DUMMYF,XINTGL)  
DIMENSION DUMMYF(50)  
H=(XMAX-XMIN)/FLOAT(N)  
SUM=0.0  
DO 4 I=2,N  
IF(MOD(I,2))2,2,3  
2 SUM=SUM+4.\*DUMMYF(I)  
GO TO 4  
3 SUM=SUM+2.\*DUMMYF(I)  
4 CONTINUE  
XINTGL=H/3.\*(DUMMYF(1)+SUM+DUMMYF(N+1))  
RETURN  
END  
C \*\*\*\*\* E N D \*\*\*\*\*